PRACTICE PAPER – 4

SOLUTIONS

SECTION – A

1. Find the chord of contact of (1, 1) to the circle $x^2 + y^2 = 9$.

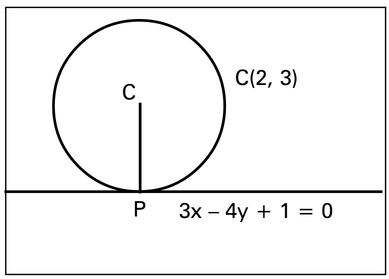
Sol. Equation of the circle is $x^2 + y^2 = 9$

Equation of the chord of contact is $x.1 + y.1 = 9$

i.e. $x + y = 9$

2. Find the equation of circle with centre (2, 3) and touching the line 3x – 4y + 1 = 0.

Sol. $CP = r = \left|\frac{3(2) - 4(3) + 1}{\sqrt{3^2 + 4^2}}\right| = 1$



Equation of circle

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y - 3)^2 = 1$

$x^2 + y^2 - 4x - 6y + 12 = 0$

3. Find the equation of the circle whose diameter is the common chord of the circles.

$x^2 + y^2 + 2x + 3y + 1 = 0$

$x^2 + y^2 + 4x + 3y + 2 = 0$.

Sol. $S \equiv x^2 + y^2 + 2x + 3y + 1 = 0$ —— (1)

$S' \equiv x^2 + y^2 + 4x + 3y + 2 = 0$ —— (2)

Here the common chord is the radical axis of (1) and (2). The equation of the radical axis is $S - S' = 0$.

i.e., $2x + 1 = 0$ —— (3)

The equation of any circle passing through the points of intersection of (1) and (3) is $(S + \lambda L = 0)$

$(x^2 + y^2 + 2x + 3y + 1) + \lambda(2x + 1) = 0$

$x^2 + y^2 + 2(\lambda + 1)x + 3y + (1 + \lambda) = 0$ — (4)

The centre of this circle is $\left(-(\lambda+1), \frac{3}{2}\right)$.

For the circle (4), $2x + 1 = 0$ is one chord. This chord will be a diameter of the circle (4) if the centre of (4) lies on (3).

$\therefore 2\{-(\lambda + 1)\} + 1 = 0$

$\Rightarrow \lambda = -\frac{1}{2}$

Thus equation of the circle whose diameter is the common chord (1) and (2)

[put $\lambda = \frac{1}{2}$ in equation (4)]

$2(x^2 + y^2) + 2x + 6y + 1 = 0$

4. Find the equation of the parabola whose vertex is (3, –2) and focus is (3, 1).

Sol. The abcissae of the vertex and focus are equal to 3. Hence the axis of the parabola is $x = 3$, a line parallel to y–axis, focus is above the vertex.

a = distance between focus and vertex = 3.

$\therefore$ Equation of the parabola

$(x - 3)^2 = 4(3)(y + 2)$

i.e., $(x - 3)^2 = 12(y + 2)$.

5. If the eccentricity of a hyperbola is $\frac{5}{4}$, then find the eccentricity of its conjugate hyperobola.

Sol. If e and e, are the eccentricity of a hyperbola and its conjugate hyperbola, then

$\frac{1}{e^2} + \frac{1}{e_1^2} = 1$

Given e $= \frac{5}{4} = \frac{16}{25} + \frac{1}{e_1^2} = 1$

$\frac{1}{e_1^2} = 1 - \frac{16}{25} = \frac{9}{25}$ $e_1^2 = \frac{25}{9} \Rightarrow e_1 = \frac{5}{3}$

6. Evaluate $\int (\tan x + \log \sec x) e^x dx$ on $\left(\left(2n - \frac{1}{2}\right)\pi, \left(2n + \frac{1}{2}\right)\pi\right), n \in z.$

Sol. $t = \log|\sec x| \Rightarrow dt = \frac{1}{\sec x}$. $\sec x. \tan x\, dx = \tan x\, dx$

$\int (\tan x + \log \sec x)\, e^x\, dx = e^x. \log|\sec x| + C$

7. Evaluate $\int \frac{dx}{(x+1)(x+2)}$.

Sol. $\int \frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$

$\int \frac{dx}{(x+1)(x+2)} = \int \frac{dx}{x+1} - \int \frac{dx}{x+2}$

$= \log|x+1| - \log|x+2| + C = \log\left|\frac{x+1}{x+2}\right| + C$

8. Evaluate $\int_0^4 (x + e^{2x})\, dx$.

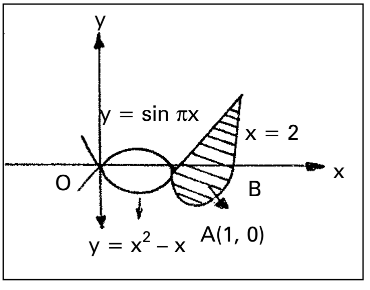
Sol. $\int_0^4 (x + e^{2x})dx = \left[\frac{x^2}{2} + \frac{e^{2x}}{2}\right]_0^4 = \frac{16}{2} + \frac{e^8}{2} - \frac{1}{2} = \frac{15 + e^8}{2}$

9. Find the area of region enclosed by the curves $y = \sin \pi x$, $y = x^2 - x$, $x = 2$.

Sol. Required area

$= \int_1^2 (x^2 - x - \sin \pi x)dx$

$= \left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{\cos \pi x}{\pi}\right)_1^2$



$$= \left[\left(\frac{8}{3}-\frac{4}{2}+\frac{1}{\pi}\right)-\left(\frac{1}{3}-\frac{1}{2}+\frac{-1}{\pi}\right)\right]$$

$$= \frac{2}{3}+\frac{1}{\pi}+\frac{1}{6}+\frac{1}{\pi} = \frac{2}{\pi}+\frac{5}{6}$$

10. Find the general solution of $x = y\frac{dy}{dx} = 0$.

Sol. Given equation is $x + y.\ \frac{dy}{dx} = 0$

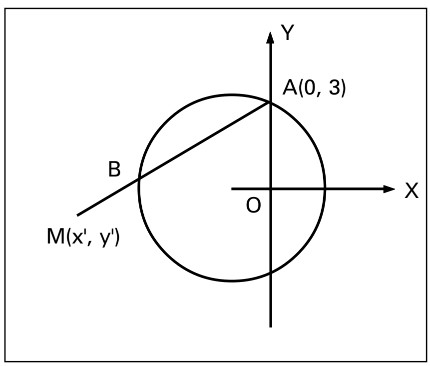
$x.\ dx + y.\ dy = 0$

Integrating $\frac{x^2}{2} + \frac{y^2}{2} = c$ or $x^2 + y^2 = 2\,c = c'$

SECTION – B

11. From the point (0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$ a chord AB is drawn and extended to a point M such that AM = 2AB. Find the equation of the locus of M.

Sol. Let $M = (x', y')$



Given that AM = 2AB

AB + BM = 2AB + AB

$\Rightarrow$ BM = AB

B is the mid point of AM

Co- ordinates of B are

$$\left(\frac{x'}{2} - \frac{y'+3}{2}\right)$$

B is a point on the circle

$$\left(\frac{x'}{2}\right)^2 + 4\left(\frac{x'}{2}\right) + \left(\frac{y'+3}{2} - 3\right)^2 = 0$$

$$\frac{x'^2}{4} + 2x' + \frac{y'^2 - 6y' + 9}{4} = 0$$

$$x'^2 + 8x' + y'^2 - 6y' + 9 = 0$$

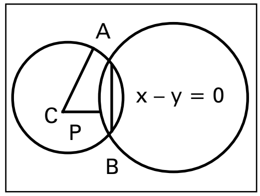
Locus of $M(x', y')$ is $x^2 + y^2 + 8x - 6y + 9 = 0$, which is a circle.

12. Find the equation and length of the common chord of the two circles $S \equiv x^2 + y^2 + 3x + 5y + 4 = 0$ and $S' \equiv x^2 + y^2 + 5x + 3y + 4 = 0$.

Sol. Equations of the given circles are

$S \equiv x^2 + y^2 + 3x + 5y + 4 = 0$ —— (1)

and $S' \equiv x^2 + y^2 + 5x + 3y + 4 = 0$ — (2)



Equations of the common chord is

$S - S' = 0$

$-2x + 2y = 0$

$L \equiv x - y = 0$

Centre of $S = 0$ is $C\left(-\frac{3}{2}, -\frac{5}{2}\right)$

$$r = \sqrt{\frac{9}{4}+\frac{25}{4}-4} = \sqrt{\frac{9+25-16}{4}} = \sqrt{\frac{18}{4}} = \sqrt{\frac{9}{2}}$$

P = length of the perpendicular from C on AB

$$= \frac{\left|-\frac{3}{2}+\frac{5}{2}\right|}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}.$$

AB = length of the common chord

$$= 2\sqrt{r^2 - p^2} = 2\sqrt{\frac{9}{2}-\frac{1}{2}} = 2\sqrt{4} = 4 \text{ units.}$$

13. Find the equation of the ellipse in the standard form such that distance between foci is 8 and distance between directrices is 32.

Sol. Distance between foci = 8.

Distance between directrices = 32.

$2ae = 8 \qquad \frac{2a}{e} = 32$

$ae = 4 \qquad \frac{a}{e} = 16$

$(ae)\left(\frac{a}{e}\right) = 64$

$a^2 = 64$

$b^2 = a^2 - a^2 e^2$

$= 64 - 16 = 48$

Equation of the ellipse is $\therefore \frac{x^2}{64} + \frac{y^2}{48} = 1$

14. Find the equation of tangent and normal to the ellipse $2x^2 + 3y^2 = 11$ at the point whose ordinate is 1.

Sol. Equation of the ellipse is $2x^2 + 3y^2 = 11$

Given $y = 1$

$2x^2 + 3 = 11 \quad \Rightarrow 2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

Points on the ellipse are P (2, 1) and Q (– 2, 1)

Case i) P (2, 1)

Equation of the tangent is $2x.\,2 + 3y.\,1 = 11$; $4x + 3y = 11$

The normal is perpendicular to the tangent

Equation of the normal at P can be taken as $3x - 4y = k$.

The normal passes through P (2, 1) $6 - 4 = k \Rightarrow k = 2$

Equation of the normal at P is $3x - 4y = 2$.

Case ii) Q (–2, 1)

Equation of the tangent at Q is $2x(-2) + 3y.1 = 11$

$-4x + 3y = 11 \Rightarrow 4x - 3y + 11 = 0$

Equation of the normal can be taken as $3x+4y = k$

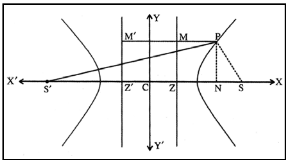
The normal passes through Q (–2, 1) $-6 + 4 = k \Rightarrow k = -2$

Equation of the normal at Q is $3x + 4y = -2$ or $3x + 4y + 2 = 0$.

15. The difference of the focal distances of any point on the hyperbola is constant.

Sol. Let P(x, y) be any point on the hyperbola whose centre is the origin C foci are S, S' directries are $\overleftrightarrow{ZM}$ and $\overleftrightarrow{Z'M'}$ as shown in fig.

Let PN, PM, PM' be the perpendiculars drawn from P upon x–axis and the two directrices respectively.



Now SP = e(PM) = e(NZ) = e(CN – CZ).

$\therefore$ $SP = e\left(x - \frac{a}{e}\right) = ex - a.$

and S'P = e(PM') = e(NZ') = e(CN + CZ')

$= e\left(x + \frac{a}{e}\right) = ex + a$

$\therefore$ S'P – SP = 2a.

By the above theorem, the hyperbola is sometimes defined as the locus of a point, the difference of whose distances from two fixed points is constant.

16. Solve $\int_0^1 x \tan^{-1} x\, dx$.

Sol. $\int_0^1 x.\tan^{-1} x\, dx$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$x = 0 \Rightarrow \theta = 0;\ x = 1 \Rightarrow \theta = \frac{\pi}{4}$

$I = \int_0^{\pi/4} \theta.\tan\theta.\sec^2\theta\, d\theta$

$= \left(\theta.\frac{\tan^2\theta}{2}\right)_0^{\pi/4} - \frac{1}{2}\int_0^{\pi/4} (\tan^2\theta)\, d\theta$

$$= \left(\frac{\pi}{8} - 0\right) - \frac{1}{2}\int_0^{\pi/4} (\sec^2\theta - 1)\, d\theta = \frac{\pi}{8} - \frac{1}{2}(\tan\theta - \theta)_0^{\pi/4}$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{1}{2}.\frac{\pi}{4} = \frac{\pi}{4} - \frac{1}{2}$$

17. Solve $\frac{dy}{dx}(x^2y^3 + xy) = 1$.

Sol. $\frac{dx}{dy} = xy + x^2y^3$

This is Bernoulli's equation $x^{-2}.\frac{dx}{dy} - \frac{1}{x}.y = y^3$

Put $z = -\frac{1}{x}$ so that $\frac{dz}{dy} = \frac{1}{x^2}\frac{dx}{dy}$ $\frac{dz}{dy} + z.y = y^3$

$$\text{I.F.} = e^{\int y\,dy} = e^{\frac{y^2}{2}}$$

$$z.e^{y^2/2} = \int y^3.e^{\frac{y^2}{2}}.dy$$

Consider $\int y^3.e^{\frac{y^2}{2}}.dy$

$$t = \frac{y^2}{2} \Rightarrow dt = y\,dy$$

$$\int y^3.e^{y^2}.dy = \int t.\,dt.\,e^t = e^t(t-1) = e^{\frac{y^2}{2}}\left(\frac{y^2}{2} - 1\right)$$

$$z.\,e^{\frac{y^2}{2}} = e^{\frac{y^2}{2}}\left(\frac{y^2}{2} - 1\right) + c$$

$$z = \frac{y^2}{2} - 1 + c.\,e^{-\frac{y^2}{2}}$$

$$-\frac{1}{x} = \frac{y^2}{2} - 1 + c.\,e^{-\frac{y^2}{2}}$$

$$-1 = x\left(\frac{y^2}{2} - 1 + c.e^{-\frac{y^2}{2}}\right)$$

Solution is $1 + x\left(\frac{y^2}{2} - 1 + c.e^{-\frac{y^2}{2}}\right) = 0$

SECTION – C

18. Find the equation of the circum circle of the traingle formed by the straight lines 5x – 3y + 4 = 0, 2x + 3y – 5 = 0, x + y = 0.

Sol. AB : $5x - 3y + 4 = 0$

AC : $2x + 3y - 5 = 0$

BC : $x + y = 0$

A : $\left(\frac{1}{7}, \frac{11}{7}\right)$ C : $(-5, 5)$

B : $\left(-\frac{1}{2}, \frac{1}{2}\right)$

AB : $5x - 3y + 4 = 0$

BC : $x + y = 0$

Equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Points A, B, C are on circumference of circle

$$\frac{1}{49} + \frac{121}{49} + \frac{2}{7}g + \frac{22}{7}f + c = 0 \quad \text{— (i)}$$

$$25 + 25 - 10g + 10f + c = 0 \quad \text{— (ii)}$$

$$\frac{1}{4} + \frac{1}{4} - g + f + c = 0 \quad \text{— (iii)}$$

(Or)

$$1 - 2g + 2f + 2c = 0$$

(ii) – (iii) we get $\left(50 - \frac{1}{2}\right) - 9g + 9f = 0$

$\frac{11}{2} - g + f = 0$ — (iv)

(iii) – (i) we get $\left(\frac{1}{2} - \frac{122}{49}\right) - g - \frac{2}{7}\ g + f - \frac{22}{7}\ f = 0$

$\frac{-195}{2 \times 49} - \frac{9}{7} g - \frac{15}{7} f = 0$

$\frac{65}{2 \times 49} + \frac{3}{7} g + \frac{5}{7} f = 0$ — (v)

Solving (iv) and (v) we get

$g = \frac{40}{14}$; $f = \frac{-37}{14}$; $c = \frac{70}{14}$

$x^2 + y^2 + \frac{80}{14} x - \frac{74}{14} y + \frac{70}{14} = 0$

Required equation of circle be

$7(x^2 + y^2) + 40x - 37y + 35 = 0$

19. Find the equation of the circum circle of the triangle formed by the line ax + by + c = 0 (abc ≠ 0) and the coordinate axes.

Sol. Let the line ax + by + c = 0 cut X, Y axes at A and B respectively co-ordinates of O are (0, 0) A are

$\left(-\frac{c}{a}, 0\right)$ B are $\left(0, -\frac{c}{b}\right)$

Suppose the equation of the required circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

This circle passes through O(0, 0)

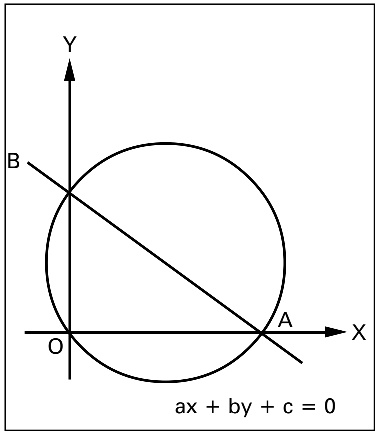
$\therefore c = 0$

This circle passes through $A\left(-\frac{c}{a}, 0\right)$

$\frac{c^2}{a^2} + 0 - 2\frac{gc}{a} = 0$

$$2g.\ \frac{c}{a} = \frac{c^2}{a^2}$$

$$\Rightarrow 2g = \frac{c}{a} \Rightarrow g = \frac{c}{2a}$$



The circle passes through

$$B\left(0, -\frac{c}{b}\right)$$

$$0 + \frac{c^2}{b^2} + 0 - 2g\ \frac{c}{b} = 0$$

$$2f\ \frac{c}{b} = \frac{c^2}{b^2}$$

$$\Rightarrow 2g = \frac{c}{b} \Rightarrow f = \frac{c}{2b}$$

Equation of the circle through O, A, B is

$$x^2 + y^2 + \frac{c}{a}x + \frac{c}{b}\ y = 0$$

$$ab(x^2 + y^2) + (bx + ay) = 0$$

This is the equation of the circum circle of ΔOAB

20. Prove that the orthocenter of the triangle formed by any three tangents to a parabola lines on the directrix of the parabola.

Sol. Let $y^2 = 4ax$ be the parabola and

$A = (at_1^2, 2at_1)$, $B = (at_2^2, 2at_2)$,

$C = (at_3^2, 2at_3)$ be any three points on it.

Now we consider the triangle PQR formed by the tangents to the parbola at A, B, C where $P = (at_1t_2, a(t_1+t_2))$,

$Q = (at_2t_3, a(t_2+t_3))$ and $R = (at_3t_1, a(t_3+t_1))$.

Equation of $\overleftrightarrow{QR}$ (i.e., the tangent at C) is

$x - t_3 y + at_3^2 = 0$.

Therefore, the attitude through P of triangle PQR is

$t_3x + y = at_1t_2\,t_3 + a(t_1 + t_2)$ —— (1)

Similarly, the attitude through Q is

$t_1x + y = at_1t_2t_3 + a(t_2+t_3)$ —— (2)

Solving (1) and (2), we get $(t_3 - t_1)$

$x = a(t_1 - t_3)$ i.e., $x = -a$.

Therefore, the orthocenter of the triangle PQR, with abscissa as –a, lies on the directrix of the parabola.

21. Find $\int \frac{2\sin x + 3\cos x + 4}{3\sin x + 4\cos x + 5}$

Sol. Let 2 sin x + 3 cos x + 4

= A(3 sin x + 4 cos x + 5) + 3(3 cos x – 4 sin x) + C

Equating the co-efficients of

sin x, we get 3A – 4B = 2

cos x, we get 4A + 3B = 3

A		B		1	
–4	✕	–2	✕	3	✕	–4
3		–3		4		3

$$\frac{A}{12+6} = \frac{B}{-8+9} = \frac{1}{9 + 16}$$

$$A = \frac{18}{25} \quad B = \frac{1}{25}$$

Equating the constants

$4 = 5A + C$

$$C = 4 - 5A = 4 - 5.\frac{18}{25} = \frac{2}{5}$$

$$\int \frac{2\sin x + 3\cos x + 4}{3\sin x + 4\cos x + 5}$$

$$= \frac{18}{25}\int dx + \frac{1}{25}\int \frac{3\cos x - 4\sin x}{3\sin x + 4\cos x + 5} + \frac{2}{5}\int \frac{dx}{3\sin x + 4\cos x + 5}$$

$$= \frac{18}{25} . x + \frac{1}{25} \log |3 \sin x + 4 \cos x + 5|$$

$$+ \frac{2}{5} \int \frac{dx}{3\sin x + 4\cos x + 5} \text{ —— (1)}$$

Let $I_1 = \int \frac{dx}{3\sin x + 4\cos x + 5}$

$t = \tan \frac{x}{2} \Rightarrow dx = \frac{2dt}{1+t^2}$

$$I_1 = \int \frac{\frac{2dt}{1+t^2}}{\frac{3-2t}{1+t^2} + \frac{4(1+t^2)}{1+t^2} + 5} = 2\int \frac{dt}{6t + 4 - 4t^2 + 5 + 5t^2}$$

$$= 2 \int \frac{dt}{t^2 + 6t + 9} = 2 \int \frac{dt}{(t+3)^2} = -\frac{2}{t+3} = -\frac{2}{3 + \tan \frac{x}{2}}$$

Substituting in (1)

$$I = \frac{18}{25} . x + \frac{1}{25} \log |3 \sin x + 4 \cos x + 5| - \frac{4}{5\left(3 + \tan\frac{x}{2}\right)} + C$$

22. **Find** $\int \frac{2x+1}{x(x^2+4)^2} dx.$

Sol. Let $\frac{2x+1}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+c}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$

$2x + 1 = A (x^2 + 4)^2 + (Bx + C) + x (x^2 + 4) + (Dx + E) x$

Equating the coefficients of like power of x, we obtain

$A + B = 0,\ C = 0,\ 8A + 4B + D = 0, 4C + E = 2, A = \frac{1}{16}$

Solving these equations, we obtain

$A = \frac{1}{16},\ B = -\frac{1}{16},\ C = 0,\ D = -\frac{1}{4}, E = 2$

$$\int \frac{2x+1}{x(x^2+4)^2} dx = \frac{1}{16}\int \frac{dx}{x} - \frac{1}{32} \int \frac{2x}{x^2+4} dx$$

$$+\int \frac{\left(-\frac{1}{4}x+2\right)}{(x^2+4)^2}\,dx \quad .... (1)$$

$\int \frac{dx}{(x^2+4)^2}$ Put $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta\, d\theta$

$$\int \frac{dx}{(x^2+4)^2} = \int \frac{2\sec^2\theta\, d\theta}{16(1+\tan^2\theta)^2} = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{8}\int \cos^2\theta\, d\theta$$

$$= \frac{1}{16}\int(1+\cos 2\theta)\, d\theta = \frac{1}{16}\left(\theta + \sin\frac{2\theta}{2}\right)$$

$$= \frac{1}{16}\left(\theta + \frac{\tan\theta}{1+\tan^2\theta}\right) + C = \frac{1}{16}\left[\tan^{-1}\left(\frac{x}{2}\right) + \frac{2x}{4+x^2}\right] + C_2$$

From (1) and (2) we get

$$\int \frac{2x+1}{x(x^2+4)^2}\,dx = \frac{1}{16}\log|x| - \frac{1}{32}\log(x^2+4) + \frac{1}{8(x^2+4)}$$

$$+ \frac{1}{8}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{4}\left(\frac{x}{4+x^2}\right) + C$$

23. **Find** $\int_0^{\pi} x\sin^7 x.\cos^6 x\, dx.$

Sol. Let $A = \int_0^{\pi} x\sin^7 x.\cos^6 x\, dx$

$$A = \int_0^{\pi} (\pi - x)\sin^7(\pi - x).\cos^6(\pi - x)\, dx$$

$$= \int_0^{\pi} (\pi - x)\sin^7 x.\cos^6 x\, dx$$

$$= \pi\int_0^{\pi} \sin^7 x.\cos^6 x\, dx - A$$

$$2A = \pi \int_0^{\pi} \sin^7 x. \cos^6 x\, dx$$

$$A = \frac{\pi}{2} \int_0^{\pi} \sin^7 x. \cos^6 x\, dx$$

$f(x) = \sin^7 x. \cos^6 x.$

$$f(\pi - x) = \sin^7 x\, (\pi - x) \cos^4 (\pi - x)$$
$$= \sin^7 x. \cos^4 x = f(x)$$

$$\int_0^{\pi} \sin^7 x. \cos^6 x\, dx = 2 \int_0^{\pi/2} \sin^7 x \cos^6 x\, dx$$

$$A = \pi \int_0^{\pi/2} \sin^7 x \cos^6 x\, dx$$

$$= \pi. \frac{6}{17}. \frac{3}{11}. \frac{1}{9}. \frac{6}{7}. \frac{4}{5}. \frac{2}{3} = \pi. \frac{16}{3003}$$

24. Solve $x \sec \left(\frac{y}{x}\right)$. (ydx + xdy) = y cosec $\left(\frac{y}{x}\right)$. (xdy – ydx).

Sol. The given equation can be written as

$$x \sec \left(\frac{y}{x}\right) \left(y + x. \frac{dy}{dx}\right)$$

$$= y \operatorname{cosec} \left(\frac{y}{x}\right) \left(x. \frac{dy}{dx} - y\right)$$

$$x. \frac{dy}{dx} \left(x. \sec \left(\frac{y}{x}\right) - y. \operatorname{cosec} \left(\frac{y}{x}\right)\right)$$

$$= -y \left[y. \operatorname{cosec} \left(\frac{y}{x}\right) + x. \sec \left(\frac{y}{x}\right)\right]$$

$$\frac{dy}{dx} = \frac{-y\left(y. \operatorname{cosec} \left(\frac{y}{x}\right) + x. \sec \left(\frac{y}{x}\right)\right)}{x\left(x. \sec \left(\frac{y}{x}\right) - y. \operatorname{cosec} \left(\frac{y}{x}\right)\right)}$$

This is a homogeneous equation.

Put y = vx

$$\frac{dy}{dx} = v + x.\frac{dv}{dx}$$

$$v + x.\frac{dv}{dx} = v\left(\frac{v\,\text{cosec}\,v + \sec v}{v\,\text{cosec}\,v - \sec v}\right)$$

$$= \frac{v\left(\frac{v}{\sin v} + \frac{1}{\cos v}\right)}{\left(v.\frac{1}{\sin v} - \frac{1}{\cos v}\right)} = \frac{v(v\cos v + \sin v)}{v\cos v - \sin v}$$

$$x.\frac{dv}{dx} = \frac{v(v\cos v + \sin v)}{v\cos v - \sin v} - v$$

$$= \frac{v(v\cos v + \sin v - v\cos v + \sin v)}{v\cos v - \sin v}$$

$$= \frac{2v\sin v}{v\cos v - \sin v}$$

$$\int \frac{v\cos v - \sin v}{v\sin v}\,dv = 2\int \frac{dx}{x}$$

$$\int \frac{\cos v}{\sin v}\,dv - \int \frac{1}{v}\,dv = 2\int \frac{dx}{x}$$

log sin v – log v = 2 log x + log c

$$\log\left(\frac{\sin v}{v}\right) = \log cx^2$$

$$\frac{\sin v}{v} = cx^2$$

$$\frac{x}{y}\sin\left(\frac{y}{x}\right) = cx^2$$

Solution is $\sin\left(\frac{y}{x}\right) = cxy$.

❖ ❖ ❖ ❖ ❖