# PRACTICE PAPER – 6

## SOLUTIONS

### SECTION – A

**I. 1. Find the value of k if the points (4, 2) and (k, –3) are conjugate points with respect to the circle $x^2 + y^2 - 5x + 8y + 6 = 0$.**

**Sol.** Equation of the circle is $x^2 + y^2 - 5x + 8y + 6 = 0$

Polar of P(4, 2) is

$$x.4 + y.2 - \frac{5}{2}(x + 4) + 4(y + 2) + 6 = 0$$

$$8x + 4y - 5x - 20 + 8y + 16 + 12 = 0$$

$$3x + 12y + 8 = 0$$

P(4, 2), Q(k, –3) are conjugate points

Polar of P passes through Q

$\therefore 3k - 36 + 8 = 0$

$3k = 28 \Rightarrow k = \frac{28}{3}$.

**2. If the circle $x^2 + y^2 + ax + by - 12 = 0$ has the centre at (2, 3) then find a, b and the radius of the circle.**

**Sol.** Equation of the circle is $x^2 + y^2 + ax + by - 12 = 0$

Centre $= \left(-\frac{a}{2}, -\frac{b}{2}\right) = (2, 3)$

$-\frac{a}{2} = 2 \qquad -\frac{b}{2} = 3$

$a = -4 \qquad b = -6$

$g = -2, f = -3, c = -12$

radius $= \sqrt{g^2 + f^2 - c} = \sqrt{4 + 9 + 12} = 5$

**3. $x^2 + y^2 + 4x + 6y - 7 = 0$, $4(x^2 + y^2) + 8x + 12y - 9 = 0$ find the equation of the radical axis of the circles.**

**Sol.** $S - S' = 0$ is radical axis.

$(x^2 + y^2 + 4x + 6y - 7)$

$$-\left(x^2 + y^2 + 2x + 3y - \frac{9}{4}\right) = 0$$

$$2x + 3y - \frac{19}{4} = 0 \Rightarrow 8x + 12y - 19 = 0$$

**4. Find the equations of axis and directrix of the parabola $y^2 + 6y - 2x + 5 = 0$.**

**Sol.** $y^2 + 6y = 2x - 5$

Adding '9' on both sides we get,

$y^2 + 6y + 9 = 2x - 5 + 9$

$[y - (-3)]^2 = 2x + 4$

$[y - (-3)]^2 = 2[x - (-2)]$

Comparing with $(y - k)^2 = 4a(x - h)$ we get,

$(h, k) = (-2, -3)$, $a = \frac{1}{2}$

Equation of the axis $y - k = 0$ i.e. $y + 3 = 0$

Equation of the directrix $x - h + a = 0$

**i.e.,** $x - (-2) + \frac{1}{2} = 0$

$2x + 5 = 0.$

**5. If the angle between the asymptotes is 30° then find its eccentricity.**

**Sol.** Angle between the asymptotes $= 2\theta = 30^\circ$

$\theta = 15^\circ$

$\tan\theta = \tan 15^\circ = \frac{b}{a}$

$$e^2 = \frac{a^2 + b^2}{a^2} = 1 + \tan^2 15^o$$

$$= \sec^2 15^o = \left(\frac{2\sqrt{2}}{\sqrt{3}+1}\right)^2 = \frac{8}{4+2\sqrt{3}} \times \frac{4-2\sqrt{3}}{4-2\sqrt{3}}$$

$$= \frac{8(4-2\sqrt{3})}{4} = 8 - 4\sqrt{3} = (\sqrt{6} - \sqrt{2})^2$$

Eccentricity = e = $\sqrt{6} - \sqrt{2}$

**6. Solve $\int \sec^2 x \operatorname{cosec}^2 x \, dx$ on $I \subset R \setminus \left(\{n\pi : n \in Z\} \cup \left\{(2n+1)\frac{\pi}{2} : n \in Z\right\}\right)$.**

**Sol.** $\int \sec^2 x . \operatorname{cosec}^2 x \, dx$

$$= \int \frac{1}{\cos^2 x \sin^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x . \sin^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x \, dx + \int \operatorname{cosec}^2 x \, dx$$

$$= \tan x - \cot x + C$$

**7. Solve $\int \frac{\cot(\log x)}{x} dx$, $x \in I \subset (0, \infty) \setminus \{e^{n\pi} : n \in Z)$.**

**Sol.** $t = \log x \Rightarrow dt = \frac{dx}{x}$

$$\int \frac{\cot(\log x)}{x} dx = \int \cot t \, dt = \log(\sin t) + C$$

$$= \log(\sin(\log x) + C$$

**8. Evaluate $\int_0^4 |2 - x|\, dx$**

**Sol.** $= \int_0^2 |2-x|\, dx + \int_2^4 |2-x|\, dx$

$= \int_0^2 (2-x)\, dx + \int_2^4 (x-2)\, dx$

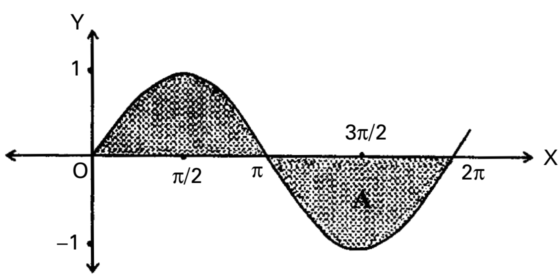
$= \left[2x - \frac{x^2}{2}\right]_0^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$

$= \left(4 - \frac{4}{2}\right) - \left[(8-8) - \left(4 - \frac{4}{2}\right)\right]$

$= 2 - 0 + 2 = 4$

**9. Find the area under the curve f(x) = sin x in [0,2π].**

**Sol.**




$f(x) = \sin x,$

We know that in $[0, \pi]$, $\sin x \geq 0$ and $[\pi, 2\pi]$, $\sin x \leq 0$

Required area $= \int_1^{\pi} \sin x\, dx + \int_{\pi}^{2\pi} (-\sin x)\, dx$

$= (-\cos x)_0^{\pi}\ [\cos x]_{\pi}^{2\pi}$

$= -\cos \pi + \cos 0 + \cos 2\pi - \cos \pi$

$= -(-1) + 1 + 1 -(-1) = 1 + 1 + 1 + 1$

$= 4.$

**10. Find the general solution of $\frac{dy}{dx} = \frac{2y}{x}$.**

**Sol.** $\frac{dy}{dx} = \frac{2y}{x}$

$\int \frac{dy}{y} = 2\int \frac{dx}{x}$

$\log c + \log y = 2 \log x$

$\log cy = \log x^2$

Solution is $cy = x^2$ where c is a constant

## SECTION – B

**II. 11. If the abscissae of points A, B are the roots of the equation, $x^2 + 2ax - b^2 = 0$ and ordinates of A, B are roots of $y^2 + 2py - q^2 = 0$, then find the equation of a circle for which $\overline{AB}$ is a diameter.**

**Sol.** Equation of the circle is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$x^2 - x(x_1 + x_2) + x_1x_2 + y^2 - y(y_1 + y_2) + y_1y_2 = 0$

$x_1, x_2$ are roots of $x^2 + 2ax - b^2 = 0$

$y_1, y_2$ are roots of $y^2 + 2py - q^2 = 0$

$x_1 + x_2 = -2a \qquad y_1 + y_2 = -2p$

$x_1x_2 = -b^2 \qquad y_1y_2 = -q^2$

$$\left[\begin{array}{c} ax^2 + bx + c = 0 \\ \text{sum of roots} = -b/a \\ \text{product} = c/a \end{array}\right]$$

Equation of circle be

$x^2 - x(-2a) - b^2 + y^2 - y(-2p) - q^2 = 0$

$x^2 + 2xa + y^2 + 2py - b^2 - q^2 = 0$

**12. If the angle between the circles**
**$x^2 + y^2 - 12x - 6y + 41 = 0$ and**
**$x^2 + y^2 + kx + 6y - 59 = 0$ is 45° find k.**

**Sol.** Suppose $\theta$ is the angle between the circles

$x^2 + y^2 - 12x - 6y + 41 = 0$

and $x^2 + y^2 + kx + 6y - 59 = 0$

$g_1 = -6, f_1 = -3, c_1 = 41,$

$g_2 = \frac{k}{2}, f_2 = 3, c_2 = -59$

$$\cos\theta = \frac{c_1 + c_2 - 2g_1 . g_2 - 2f_1 . f_2}{2r_1 r_2}$$

$$\cos 45^\circ = \frac{41 - 59 - 2(-6)\frac{k}{2} - 2(-3).3}{2\sqrt{36+9-41}\ \sqrt{\frac{k^2}{4} + 9 + 59}}$$

$$\frac{1}{\sqrt{2}} = \frac{-18 + 6k + 18}{2.2.\sqrt{\frac{k^2}{4} + 68}}$$

$$\frac{1}{\sqrt{2}} = \frac{6k}{4.\sqrt{\frac{k^2}{4} + 68}}$$

Squaring and cross - multiplying

$$4\left(\frac{k^2}{4} + 68\right) = 18k^2$$

$$\frac{2\left[k^2 + 272\right]}{4} = 9k^2$$

$k^2 + 272 = 18\ k^2$

$17k^2 = 272$

$$k^2 = \frac{272}{17} = k^2 = 16$$

$$k = \pm 4.$$

**13. Find the equation of the tangent and normal to the ellipse $9x^2 + 16y^2 = 144$ at the end of the latus rectum in the first quadrant.**

**Sol.** Given ellipse is $9x^2 + 16y^2 = 144$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{16-9}{16}} = \frac{\sqrt{7}}{4}$$

End of the latus rectum in first quadrant

$$P\left(ae, \frac{b^2}{a}\right) = \left(\sqrt{7}, \frac{9}{4}\right)$$

Equation of the tangent at P is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

$$x.\frac{\sqrt{7}}{16} + \frac{y}{9}.\left(\frac{9}{4}\right) = 1$$

$$\frac{\sqrt{7}x}{16} + \frac{y}{4} = 1 \text{ or } \sqrt{7}x + 4y = 16$$

Equation of the normal at P is

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$$

$$\frac{16x}{\sqrt{7}} - \frac{9y}{\left(\frac{9}{4}\right)} = 16 - 9$$

$$\frac{16x}{\sqrt{7}} - 4y = 7$$

$$16x - 4\sqrt{7}y = 7\sqrt{7}.$$

**14. If e, $e_1$ are the eccentricities of a hyperbola and its conjugate hyperbola prove that $\frac{1}{e^2}+\frac{1}{e_1^2} = 1$.**

**Sol.** Equation of the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$

$\therefore\ b^2 = a^2(e^2 - 1) \Rightarrow e^2 - 1 = \frac{b^2}{a^2}$

$e^2 = 1 + \frac{b^2}{a^2} = \frac{a^2+b^2}{a^2}$

$\therefore\ \frac{1}{e^2} = \frac{a^2}{a^2+b^2}$ —— (1)

Equation of the conjugate hyperbola is

$\frac{x^2}{a^2}-\frac{y^2}{b^2} = -1 \Rightarrow \frac{y^2}{b^2}-\frac{x^2}{a^2} = 1$

$\Rightarrow a^2 = b^2 ({e_1}^2 - 1) \Rightarrow e_1^2 - 1 = \frac{a^2}{b^2}$

$e_1^2 = 1 + \frac{a^2}{b^2} = \frac{a^2+b^2}{b^2}$

$\frac{1}{e_1^2} = \frac{b^2}{a^2+b^2}$ —— (2)

Adding (1) and (2)

$$\frac{1}{e^2}+\frac{1}{e_1^2} = \frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2} = \frac{a^2+b^2}{a^2+b^2} = 1.$$

**15. Solve $\int \frac{dx}{4+5\sin x}$**

**Sol.** $t = \tan\frac{x}{2} \Rightarrow dt = \sec^2\frac{x}{2} \cdot \frac{1}{2}\,dx$

$$dx = \frac{2dt}{\sec^2\frac{x}{2}} = \frac{2dt}{1+t^2}$$

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2t}{1+t^2}$$

$$I = 2\int \frac{\frac{dt}{1+t^2}}{4+5\frac{2t}{1+t^2}} = 2\int \frac{dt}{4+4t^2+10t}$$

$$= \frac{1}{2}\int \frac{dt}{t^2+\frac{5t}{2}+1} = \frac{1}{2}\int \frac{dt}{\left(t+\frac{5}{4}\right)^2-\left(\frac{3}{4}\right)^2}$$

$$= \frac{1}{2}\cdot\frac{1}{2.\frac{3}{4}}\log\left|\frac{t+\frac{5}{4}-\frac{3}{4}}{t+\frac{5}{4}+\frac{3}{4}}\right| + C$$

$$= \frac{1}{3}\log\left|\frac{4t+2}{4t+8}\right| + C = \frac{1}{3}\log\left|\frac{2t+1}{2t+4}\right| + C$$

$$= \frac{1}{3}\log\left|\frac{2\tan\frac{x}{2}+1}{2\left(\tan\frac{x}{2}\right)+2}\right| + C$$

**16. Evaluate $\int_0^{\pi/4} \log(1+\tan x)\,dx$**

**Sol.** $I = \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$

$$= \int_0^{\pi/4} \log\left(1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log\left(\frac{1+\tan x+1-\tan x}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} [\log 2 - \log(1+\tan x)\, dx$$

$$= \int_0^{\pi/4} \log 2\, dx - \int_0^{\pi/4} \log(1+\tan x)\,]\, dx$$

$$= \log 2\,(x)_0^{\pi/4} - I$$

$$2I = \frac{\pi}{4}\log 2$$

$$I = \frac{\pi}{8}\log 2$$

**17. Evaluate $\frac{dy}{dx} = \tan^2(x+y)$.**

**Sol.** $\frac{dy}{dx} = \tan^2(x+y)$

put $v = x + y$

$$\frac{dv}{dx} = 1 + \frac{dy}{dx} = 1 + \tan^2 v = \sec^2 v$$

$$\int \frac{dv}{\sec^2 v} = \int dx$$

$$= \int \cos^2 v\, .\, dv = x + c$$

$$\int \frac{(1+\cos 2v)}{2}\, dv = x + c$$

$$\int (1 + \cos 2v)\, dv = 2x + 2c$$

$$v + \frac{\sin 2v}{2} = 2x + 2c$$

$$2v + \sin 2v = 4x + c'$$

$$2(x+y) + \sin 2(x+y) = 4x + c'$$

$$x - y - \frac{1}{2}\sin[2(x+y)] = c$$

## SECTION – C

**III. 18. If (2, 0), (0, 1), (4, 5) and (0, c) are concyclic and then find c.**

**Sol.** $x^2 + y^2 + 2gx + 2fy + c_1 = 0$

Satisfies (2, 0), (0, 1) (4, 5) we get

$4 + 0 + 4g + c_1 = 0$ — (i)

$0 + 1 + 2g.\,0 + 2f + c_1 = 0$ — (ii)

$16 + 25 + 8g + 10f + c_1 = 0$ — (iii)

(ii) – (i) we get

$-3 - 4g + 2f = 0$

$4g - 2f = -3$ — (iv)

(ii) – (iii) we get

$-40 - 8g - 8f = 0$ (or)

$g + f = -5$ — (v)

Solving(iv) and (v) we get

$g = -\frac{13}{6}, f = -\frac{17}{6}$

Substituting g and f value in equation (i) we get

$4 + 4\left(-\frac{13}{6}\right) + c_1 = 0$

$c_1 = \frac{14}{3}$

Now equation $x^2 + y^2 - \frac{13}{3}x - \frac{17}{3}y + \frac{14}{3} = 0$

Now circle passes through (0, c) then

$c^2 - \frac{17}{3}c + \frac{14}{3} = 0$

$3c^2 - 17c + 14 = 0$

$\Rightarrow (3c - 14)(c - 1) = 0$

(or)

$c = 1$ or $\frac{14}{3}$.

**19. Show that the locus of the point of intersection of the lines $x \cos \alpha + y \sin \alpha = a$, $x \sin \alpha - y \cos \alpha = b$ ($\alpha$ is a parameter) is a circle.**

**Sol.** Equations of the given lines are

$x \cos \alpha + y \sin \alpha = a$

$x \sin \alpha - y \cos \alpha = b$

Let p $(x_1, y_1)$ be the point of intersection

$x_1 \cos \alpha + y_1 \sin \alpha = a$ —— (1)

$x_1 \sin \alpha - y_1 \cos \alpha = b$ —— (2)

Squaring and adding (1) and (2)

$(x_1\cos \alpha + y_1\sin \alpha)^2 + (x_1\sin \alpha - y_1\cos \alpha)^2 = a^2 + b^2$

$x_1^2 \cos^2 \alpha + y_1^2 \sin^2 \alpha + 2x_1y_1$
$\cos \alpha \sin \alpha + x_1^2 \sin^2 \alpha + y_1^2 \cos^2 \alpha$

$- 2x_1y_1 \cos \alpha \sin \alpha = a^2 + b^2$

$x_1^2 (\cos^2 \alpha + \sin^2 \alpha) + y_1^2 (\sin^2 \alpha + \cos^2 \alpha) = a^2 + b^2$

$x_1^2 + y_1^2 = a^2 + b^2$

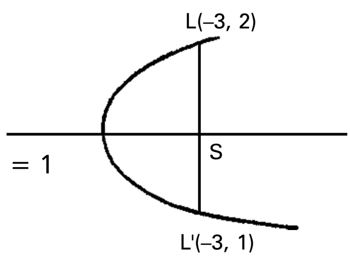
Locus of $p(x_1, y_1)$ is the circle

$$x^2 + y^2 = a^2 + b^2$$

**20. Find the equation of the parabola whose latus rectum is the line segment of joining the points (–3, 2) and (–3, 1).**

**Sol.** L (–3, 2) and L' (–3, 1) are the ends of the latus rectum.

S is the midpoint of LL'




Co-ordinates of S are $\left(-3, \frac{3}{2}\right)$

$LL' = \sqrt{(-3+3)^2 + (2-1)^2} = \sqrt{0+1} = 1$

$4|a| = 1, \Rightarrow |a| = \frac{1}{4} \Rightarrow a = \pm \frac{1}{4}$

**Case (i)** $a = -\frac{1}{4}$

Co-ordinates of A are $\left[-3+\frac{1}{4},\frac{3}{2}\right]$

Equation of the parabola is

$$\left(y-\frac{3}{2}\right)^2=-\left(x+3-\frac{1}{4}\right)$$

$$\Rightarrow \frac{(2y-3)^2}{4}=\frac{-(4x+12-1)}{4}$$

$\Rightarrow (2y-3)^2 = -(4x + 11)$

**Case (ii)** $a = \frac{1}{4}$

Co-ordinates of A are $\left[-3,-\frac{1}{4},\frac{3}{2}\right]$

Equation of the parabola is

$$\left(y-\frac{3}{2}\right)^2=\left(x+3+\frac{1}{4}\right)$$

$$\frac{(2y-3)^2}{4}=\frac{(4x+12+1)}{4}$$

i.e., $(2y-3)^2 = 4x + 13.$

**21. Evaluate** $\int \frac{1}{(x-a)(x-b)(x-c)}dx$

**Sol.** Let $\frac{1}{(x-a)(x-b)(x-c)} \equiv \frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$

$$\Rightarrow \frac{1}{(x-a)(x-b)(x-c)}$$

$$=\frac{A(x-b)(x-c)+B(x-a)(x-c)+C(x-a)(x-b)}{(x-a)(x-b)(x-c)}$$

$\Rightarrow 1 = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$ ——(1)

Put $x = a$, we get

$1 = A(a-b)(a-c) \Rightarrow A = \frac{1}{(a-b)(a-c)}$

Put x = b, we get

1 = A(0) + B(b – a) (b – c) + C(0)

$\Rightarrow B = \dfrac{1}{(b-a)(b-c)}$

Similarly $C = \dfrac{1}{(c-a)(c-b)}$

$\therefore \dfrac{1}{(x-a)(x-b)(x-c)}$

$= \dfrac{\frac{1}{(a-b)(a-c)}}{x-a} + \dfrac{\frac{1}{(b-a)(b-c)}}{x-b} + \dfrac{\frac{1}{(c-a)(c-b)}}{x-c}$

$\therefore \int \dfrac{1}{(x-a)(x-b)(x-c)}\,dx$

$= \dfrac{1}{(a-b)(a-c)} \int \dfrac{1}{x-a}\,dx + \dfrac{1}{(b-a)(b-c)} \int \dfrac{1}{x-b}\,dx$

$+ \dfrac{1}{(c-a)(c-b)} \int \dfrac{1}{x-c}\,dx$

$= \dfrac{1}{(a-b)(a-c)} \log|x-a| + \dfrac{1}{(b-a)(b-c)} \log|x-b|$

$+ \dfrac{1}{(c-a)(c-b)} \log|x-c| + k$

**22. Evaluate $\int \dfrac{\cos x + 3\sin x + 7}{\cos x + \sin x + 1}\,dx$.**

**Sol.** Let cosx + 3 sin x + 7

= A (cos x + sin x + 1)' + B(cos x + sin x + 1) + C

Comparing the coefficients

A + B = 1, A – B = 3, B + C = 7

A = –1, B = 2, C = 5

$$\int \frac{\cos x + 3\sin x + 7}{\cos x + \sin x + 1}dx = -\int \frac{-\sin x + \cos x}{\cos x + \sin x + 1}dx$$

$$+ 2\int dx + 5\int \frac{1}{\cos x + \sin x + 1}dx$$

$$= -\log|\cos x + \sin x + 1| + 2x + 5I \quad ...(1)$$

$$I = \int \frac{1}{\cos x + \sin x + 1}dx = \int \frac{dx}{2\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \frac{1}{2}\int \frac{\sec^2\frac{x}{2}dx}{1 + \tan\frac{x}{2}} = \int \frac{dt}{1+t} \quad \left(t = \tan\frac{x}{2}\right)$$

$$= \log|1 + t| = \log\left(1 + \tan\frac{x}{2}\right)$$

Substituting in $\int \frac{\cos x + 3\sin x + 7}{\cos x + \sin x + 1}dx$

$$= -\log|\cos x + \sin x + 1| + 2x + 5\log\left|1 + \tan\frac{x}{2}\right| + C$$

**23. Solve** $\int_4^9 \frac{dx}{\sqrt{(9-x)(x-4)}}$

**Sol.** Put $x = 4\cos^2\theta + 9\sin^2\theta$

$dx = (9 - 4)\sin 2\theta\, d\theta$

$dx = 5\sin 2\theta\, d\theta$

**U.L.**

$x = 4\cos^2\theta + 9\sin^2\theta$

$9 = 4\cos^2\theta + 9\sin^2\theta$

$5\cos^2\theta = 0$

$\theta = \frac{\pi}{2}$

**L.L**

$x = 4\cos^2\theta + 9\sin^2\theta$

$4 = 4\cos^2\theta + 9\sin^2\theta$

$5\sin^2\theta = 0$

$\theta = 0$

$9 - x = 9 - (4\cos^2\theta + 9\sin^2\theta) = (9-4)\cos^2\theta = 5\cos^2\theta$

$x - 4 = 4\cos^2\theta + 9\sin^2\theta - 4 = (9-4)\sin^2\theta = 5\sin^2\theta$

Let $I = \int_4^9 \frac{1}{\sqrt{(9-x)(x-4)}}dx$

$$I = \int_0^{\pi/2} \frac{1}{\sqrt{5\cos^2\theta\, 5\sin^2\theta}} 5(2\sin\theta\cos\theta)d\theta$$

$$= \frac{10}{5}\int_0^{\pi/2} \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}d\theta = \frac{10}{5}.\int_0^{\pi/2} 1d\theta = \frac{10}{5}(0)_0^{\pi/2}$$

$$= \frac{10}{5}.\frac{\pi}{2} = \frac{10}{10}\pi = \pi$$

**24. Solve $\frac{dy}{dx} = \frac{3y-7x+7}{3x-7y-3}$**

**Sol.** Let x = x + h, y = y + k so that $\frac{dy}{dx} = \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{3(y+k)-7(x+h)+7}{3(x+h)-7(y+k)-3} = \frac{3y-7x+(3k-7h+7)}{(3x-7y)+(3h-7k-3)}$$

$-7h + 3k - 3 = 0$

$3h - 7k + 7 = 0$

| | h | | k | | I | |
|---|---|---|---|---|---|---|
| +3 | ╳ | –3 | ╳ | –7 | ╳ | 3 |
| –7 | | 7 | | 3 | | –7 |

$$\frac{h}{21-21} = \frac{k}{-9+49} = \frac{1}{+49-9}$$

h = 0 and k = 1

$$\frac{dy}{dx} = \frac{3y-7x}{3x-7y}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x.\frac{dv}{dx}$

$$v + x.\frac{dv}{dx} = \frac{x(3v-7)}{x(3-7v)}$$

$$x.\frac{dv}{dx} = \frac{3v-7}{3-7v} - v = \frac{3v-7-3v+7v^2}{3-7v} = \frac{7v^2-7}{3-7v} = \frac{7v^2-7}{3-7v}$$

$$\frac{3-7v}{7v^2-7} = \frac{dx}{x}$$

$$\int\frac{3}{7v^2-7}dv - \int\frac{7v\,dv}{7v^2-7} = \int\frac{dx}{x}$$

$$ln\,x = \frac{3}{14}\,ln\left|\frac{v-1}{v+1}\right| - \frac{1}{2}\,ln\,|v^2-1|$$

14 log x – log c

$$x = 3\log\left|\frac{v-1}{v+1}\right| - 7\log|v^2-1|$$

$\Rightarrow 14\,ln\,x - ln\,c$

$= 3ln\,(v-1) - 3ln\,(v+1) - 7ln\,(v+1) - 7ln\,(v-1)$

$14ln\,x - ln\,c = -10\,ln\,(v+1) - 4\,ln\,(v-1)$

$ln\,(v+1)^5 + ln\,(v-1)^2 + ln\,x^7 = ln\,c$

$(v+1)^5.(v-1)^2.x^7 = c$

$$\left(\frac{y}{x}+1\right)^5\left(\frac{y}{x}-1\right)^2.x^7 = c$$

$(y-x)^2(y+x)^5 = c$

$[y-(x-1)]^2(y+x-1)^5 = c$

Solution is $[y-x+1]^2(y+x-1)^5 = c$.

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