SOLUTIONS FOR PRACTICE PAPER - 3

SECTION – A

I. 1. Find the equation of the straight line passing through the point (–2, 4) and making non-zero intercepts on the axis of coordinates whose sum is zero.

Sol. Let x – intercept = a

y – intercept = b

Given that $a + b = 0 \Rightarrow b = -a$

Intercept form $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{a} + \frac{y}{-a} = 1$$

$$\Rightarrow x - y = a$$

If this line passing through the point (–2, 4) then

$$-2 - 4 = a$$

$$\Rightarrow a = -6$$

Hence required straight line equation is

$$x - y = -6$$

$$\Rightarrow x - y + 6 = 0.$$

2. Find the value of P if the straight lines x + P = 0, y + 2 = 0, 3x + 2y + 5 = 0 are concurrent.

Sol. Given straight line equations are

$x + P = 0$ ——— (1)

$y + 2 = 0 \Rightarrow y = -2$ ——— (2)

$3x + 2y + 5 = 0$ ——— (3)

Solving (2) & (3)

$3x + 2(-2) + 5 = 0 \Rightarrow 3x + 1 = 0$

$$\Rightarrow x = \frac{-1}{3}$$

Point of intersection of (2) & (3) is $\left(\frac{-1}{3}, -2\right)$

Since (1), (2), (3) are concurrent

This point lies on (1)

$\Rightarrow \frac{-1}{3} + P = 0$

$\Rightarrow P = \frac{1}{3}$

3. Find the ratio in which the XZ-plane divides the line joining A(–2, 3, 4) and B(1, 2, 3).

Sol. Given A = (–2, 3, 4)

B = (1, 2, 3)

XZ - Plane divides the line joining AB in the ratio

$= -y_1 : y_2$

$= -3 : 2$

$= 3 : 2$ externally

4. Find the direction cosines of the normal to the plane $x + 2y + 2z - 4 = 0$.

Sol. Given plane equation is

$x + 2y + 2z - 4 = 0$

$\Rightarrow x + 2y + 2z = 4$

$$\Rightarrow \frac{x}{\sqrt{1^2 + 2^2 + 2^2}} + \frac{2}{\sqrt{1^2 + 2^2 + 2^2}}y + \frac{2}{\sqrt{1^2 + 2^2 + 2^2}}Z = \frac{4}{\sqrt{1^2 + 2^2 + 2^2}}$$

$$\Rightarrow \frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z = \frac{4}{3}$$

$\therefore$ The direction cosines of the normal to the plane are

$$\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$$

5. Compute $\lim_{x \to 0}\left(\frac{e^{3x}-1}{x}\right)$.

Sol. $\lim_{x \to 0} \frac{e^{3x}-1}{x} = 3 \lim_{x \to 0} \frac{e^{3x}-1}{3x}$

$= 3(1)$

$= 3$

$\therefore \lim_{x \to 0} \frac{e^{3x}-1}{x} = 3$

6. Evaluate $\lim_{x \to 0}\left(\frac{1-\cos 2mx}{\sin^2 nx}\right)$ (m. n ∈ Z).

Sol. $\lim_{x \to 0} \frac{1-\cos 2mx}{\sin^2 nx} = \lim_{x \to 0} \frac{2\sin^2 mx}{\sin^2 nx}$

$$= 2 \lim_{x \to 0} \frac{\frac{\sin^2 mx}{x^2}}{\frac{\sin^2 nx}{x^2}}$$

$$= 2 \frac{\left[\lim_{x \to 0} \frac{\sin mx}{x}\right]^2}{\left[\lim_{x \to 0} \frac{\sin nx}{x}\right]^2} = \frac{2m^2}{n^2}$$

7. Find the derivative of $\tan^{-1}$ (log x).

Sol. Let y = Tan^{-1} (log x)

differentating w.r. to 'x' on bothsides, we have

$$\frac{dy}{dx} = \frac{1}{1+(\log x)^2} \cdot \frac{d}{dx} (\log x)$$

$$= \frac{1}{1+(\log x)^2} \cdot \frac{1}{x}$$

$$= \frac{1}{x\left[1+(\log x)^2\right]}$$

8. Find the derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Sol. Let y = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

put x = Tan θ ⇒ θ = $\mathrm{Tan}^{-1}x$

$= \sin^{-1}\left(\frac{2\,\mathrm{Tan}\theta}{1+\mathrm{Tan}^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta$

$= 2\,\mathrm{Tan}^{-1}x$

Differentiating w.r.to 'x' on bothsides, we have

$\frac{dy}{dx} = 2\frac{d}{dx}(\mathrm{Tan}^{-1}x)$

$= 2.\frac{1}{1+x^2} = \frac{2}{1+x^2}$

9. Find dy and Δy of y = f(x) = x^2 + x at x = 10 when Δx = 0.1

Sol. Given f(x) = $x^2 + x$, x = 10 and Δx = 0.1

$dy = f'(x)\,\Delta x$

$= (2x + 1)\,\Delta x$

$= [2(10) + 1]\,(0.1)$

$= (21)\,(0.1)$

$= 2.1$

$\Delta y = f(x + \Delta x) - f(x)$

$= (x + \Delta x)^2 + (x + \Delta x) - (x^2 + x)$

$= x^2 + 2x\,\Delta x + (\Delta x)^2 + x + \Delta x - x^2 - x$

$= 2x\,\Delta x + (\Delta x)^2 + \Delta x$

$= 2(10)\,(0.1) + (0.1)^2 + 0.1$

$= 2 + 0.01 + 0.1$

$= 2.11$

10. Verify Rolle's theorem for the function $f(x) = x^2 + 4$ in $[-3, 3]$.

Sol. Given $f(x) = x^2 + 4$

Since f is a second degree polynomial

$\therefore$ f is continuous on $[-3, 3]$ and f is desivable on $(-3, 3)$

Also $f(-3) = 9 + 4 = 13$

$f(3) = 9 + 4 = 13$

$\therefore f(-3) = f(3)$

$\therefore$ f satisfies all the conditions of Role's theorum.

$\therefore$ There exists $c \in (-3, 3)$ such that $f^1(c) = 0$

But $f^1(x) = 2x$

$f^1(c) = 2c$

$0 = 2c$

$\Rightarrow c = 0 \in (-3, 3)$

Hence Rolle's theorum is verified.

SECTION – B

II. 11. If the distance from the Point P to the Points (2, 3) and (2, –3) are in the ratio 2 : 3, then find the equation of the Locus of P.

Sol. Let A = (2, 3) and B = (2, –3)

Let $p(x_1, y_1)$ be any point on the locus

Given geometric condition is PA : PB = 2 : 3

$$\Rightarrow \frac{PA}{PB} = \frac{2}{3}$$

$$\Rightarrow 3PA = 2PB$$

$$\Rightarrow 9PA^2 = 4PB^2$$

$$\Rightarrow 9[(x_1 - 2)^2 + (y_1 - 3)^2] = 4[(x_1 - 2)^2 + (y_1 + 3)^2]$$

$$\Rightarrow 9\left[x_1^2 - 4x_1 + 4 + y_1^2 - 6y_1 + 9\right]$$

$$= 4\left[x_1^2 - 4x_1 + 4 + y_1^2 + 6y_1 + 9\right]$$

$\Rightarrow 9x_1^2 + 9y_1^2 - 36x_1 - 54y_1 + 117 = 4x_1^2 + 4y_1^2 - 16x_1 + 24y_1 + 52$

$\Rightarrow 5x_1^2 + 5y_1^2 - 20x1 - 78y1 + 65 = 0$

$\therefore$ Locus of P is $5x^2 + 5y^2 - 20x - 78y + 65 = 0$.

12. When the axes are rotated through an angle 45°, the transformed equation of a curve is $17x^2 - 16xy + 17y^2 = 225$. Find the original equation of the curve.

Sol. Angle of rotation $\theta = 45°$

$x = x\cos\theta + y\sin\theta$

$= 2\cos 45° + y\sin 45°$

$= x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{x+y}{\sqrt{2}}$

$y = -x\sin\theta + y\cos\theta$

$= -x\sin 45° + y\cos 45°$

$= -x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{-x+y}{\sqrt{2}}$

The original equation of $17x^2 - 16xy + 17y^2 = 225$ is

$$17\left(\frac{x+y}{\sqrt{2}}\right)^2 - 16\left(\frac{x+y}{\sqrt{2}}\right)\left(\frac{-x+y}{\sqrt{2}}\right) + 17\left(\frac{-x+y}{\sqrt{2}}\right)^2 = 225$$

$$\Rightarrow 17\left(\frac{x^2+2xy+y^2}{2}\right) - 16\left(\frac{y^2-x^2}{2}\right) + 17\left(\frac{x^2-2xy+y^2}{2}\right) = 225$$

$\Rightarrow 17x^2 + 34xy + 17y^2 - 16y^2 + 16x^2 + 17x^2 - 34xy + 17y^2 = 450$

$\Rightarrow 50x^2 + 18y^2 = 450$

$\Rightarrow 25x^2 + 9y^2 = 225$

13. Find the equation of the straight line passing through the points (–1, 2) and (5, –1) and also find the area of the triangle formed by it with the axes of coordinates.

Sol. Let A = (–1, 2)

B = (5, –1)

Equation of the straight line passing through the points A and B is

$$Y - Y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow y - 2 = \frac{-1-2}{5+1}(x + 1)$$

$$\Rightarrow 6y - 12 = -3x - 3$$

$$\Rightarrow 3x + 6y - 9 = 0$$

$$\Rightarrow x + 2y - 3 = 0$$

$$\Rightarrow x + 2y = 3$$

$$\Rightarrow \frac{x}{3} + \frac{y}{3/2} = 1$$

x – intercept = 3, y – intercept = 3/2

$$\text{Area of } \Delta OAB = \frac{1}{2}\left|(x - \text{intercept})\ (y - \text{intercept})\right|$$

$$= \frac{1}{2}\left|(3)\left(\frac{3}{2}\right)\right|$$

$$= \frac{9}{4} \text{ sq.units.}$$

14. Check the continuity of the following function at 2 :

$$f(x) = \begin{cases} \frac{1}{2}(x^2 - 4) & \text{if } 0 < x < 2 \\ 0 & \text{if } x = 2 \\ 2 - 8x^{-3} & \text{if } x > 2 \end{cases}$$

Sol. Given $f(x) = \begin{cases} \frac{1}{2}(x^2 - 4) & \text{if } 0 < x < 2 \\ 0 & \text{if } x = 2 \\ 2 - 8x^{-3} & \text{if } x > 2 \end{cases}$

$$\lim_{x \to 2-} f(x) = \lim_{x \to 2-} \frac{1}{2}(x^2 - 4)$$

$$= \frac{1}{2} \lim_{h \to 0} [(2 - h)^2 - 4]$$

$$= \frac{1}{2}(4 - 4)$$

$$= 0$$

$$\lim_{x \to 2+} f(x) = \lim_{x \to 2+} \left(2 - \frac{8}{2^3}\right)$$

$$= \lim_{h \to 0} \left(2 - \frac{8}{(2+h)^3}\right)$$

$$= 2 - \frac{8}{8}$$

$$= 2 - 1$$

$$= 1$$

$$\therefore \lim_{x \to 2-} f(x) \neq \lim_{x \to 2+} f(x)$$

$\therefore$ f is discontinuous at $x = 2$

15. Find the derivative of the function sin 2x from the first principle.

Sol. Let $f(x) = \sin 2x$

By first principle

$$f^1(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin 2(x+h) - \sin 2x}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{2x + 2h + 2x}{2}\right)\sin\left(\frac{2x + 2h - 2x}{2}\right)}{h}$$

$$= \lim_{h \to 0} 2\cos(2x + h) \lim_{h \to 0} \frac{\sin h}{h}$$

$$= 2\cos(2x + 0).1$$

$$= 2\cos 2x$$

$\therefore f^1x = 2\cos 2x.$

16. Find the equation of tangent and normal to the curve $y = x^3 + 4x^2$ at (–1, 3).

Sol. Given curve equation is $y = x^3 + 4x^2$ ——— (1)

$$\frac{dy}{dx} = 3x^2 + 8x$$

$$m = \left(\frac{dy}{dx}\right)(-1, 3) = 3(-1)^2 + 8(-1)$$

$$= 3 - 8$$

$$= -5$$

The equation of the tangent to the curve (1) at (–1, 3) is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -5(x + 1)$$

$$y - 3 = -5x - 5$$

$$\Rightarrow 5x + y + 2 = 0$$

The equation of the normal to the curve (1) at (–1, 3) is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$\Rightarrow y - 3 = \frac{-1}{-5}(x + 1)$$

$$\Rightarrow 5y - 15 = x + 1$$

$$\Rightarrow x - 5y + 16 = 0$$

17. The volume of a cube is increasing at a rate of 9 $(\text{centimetres})_3$ per second. How fast is the surface area increasing when the length of the edge is 10 centimeters ?

Sol. Let x be the length of the edge of the cube, v be its volume and s be its surface area.

Given $\frac{dv}{dt} = 9 \text{ cm}^3/\text{sec}$

Since $v = x^3$

$$\frac{dv}{dt} = 3x^2 \frac{dx}{dt}$$

$$9 = 3x^2 \frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{3}{x^2}$$

Since $s = 6x^2$

$$\frac{ds}{dt} = 12x \frac{dx}{dt}$$

$$= 12x \left(\frac{3}{x^2}\right)$$

$$= \frac{36}{x}$$

When $x = 10$

$$\frac{ds}{dt} = \frac{36}{10} = 3.6 \text{ cm}^2/\text{sec}$$

SECTION – C

III.18. Find the circumcenter of the triangle whose vertices are (–2, 3), (2, –1) and (4, 0).

Sol. Let A = (–2, 3)

B = (2, –1)

C = (4, 0)

Let $s(\alpha, \beta)$ be the circum center of the ΔABC

Then SA = SB = SC

SA = SB

$\Rightarrow SA^2 = SB^2$

$\Rightarrow (\alpha + 2)^2 + (\beta - 3)^2 = (\alpha - 2)^2 + (\beta + 1)^2$

$\Rightarrow \alpha^2 - 4\alpha + 4 + \beta^2 + 6\beta + 9 = \alpha^2 - 4\alpha + 4 + \beta^2 + 2\beta + 1$

$\Rightarrow 8\alpha - 8\beta + 8 = 0$

$\Rightarrow \alpha - \beta + 1 = 0$ ——— (1)

SB = SC

$\Rightarrow SB^2 = SC^2$

$\Rightarrow (\alpha - 2)^2 + (\beta + 1)^2 = (\alpha - 4)^2 + (\beta - 0)^2$

$\Rightarrow \alpha^2 - 4\alpha + 4 + \beta^2 + 2\beta + 1 = \alpha^2 - 8\alpha + 16 + \beta^2$

$\Rightarrow 4\alpha + 2\beta - 11 = 0$ ——— (2)

Solving (1) & (2)

	α		β		1	
–1		1		1		–1
2		–11		4		2

$$\frac{\alpha}{11-2} = \frac{\beta}{4+11} = \frac{1}{2+4}$$

$$\frac{\alpha}{9} = \frac{\beta}{15} = \frac{1}{6}$$

$$\Rightarrow \alpha = \frac{3}{2}, \beta = \frac{5}{2}$$

$\therefore$ Circumcenter, S = $\left(\frac{3}{2}, \frac{5}{2}\right)$

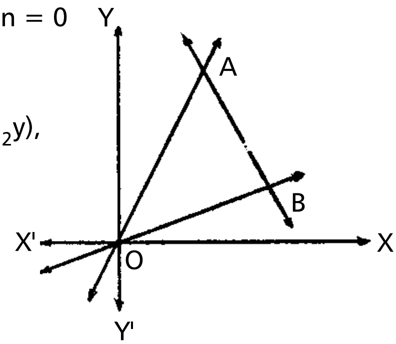
19. Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is

$$\left|\frac{n^2\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2}\right|.$$

Sol. Let $\overleftrightarrow{OA}$ and $\overleftrightarrow{OB}$ be the pair of straight lines represented by the equation

$ax^2 + 2hxy + by^2 = 0$ (see figure)

and $\overline{AB}$ be the line $lx + my + n = 0$



Let $ax^2 + 2hxy + by^2$

$$\equiv (l_1x + m_1y)(l_2x + m_2y),$$

and $\overline{OA}$ and $\overline{OB}$ be the lines.

$l_1x + m_1y = 0$ and

$l_2x + m_2y = 0$ respectively.

Let A = (x_1, y_1) and B = (x_2, y_2).

Then $l_1x_1 + m_1y_1 = 0$ and $lx_1 + my_1 + n = 0$.

So, by the rule of cross–multiplication, we obtain

$\frac{x_1}{m_1 n} \quad \frac{y_1}{nl_1} \quad \frac{1}{l_1 m \quad lm_1}$ and therefore

$$x_1 = \frac{m_1 n}{l_1 m \quad lm_1} ; y_1 = \frac{nl_1}{l_1 m \quad lm_1}$$

Similarly $x_2 = \frac{m_2 n}{l_2 m \quad lm_2}$; $y_2 = \frac{nl_2}{l_2 m \quad lm_2}$

$$\therefore \text{ Area of } \Delta \text{ OAB } = \frac{1}{2} |x_1y_2 - x_2y_1|$$

$$= \frac{1}{2}\left|\frac{n^2(l_1m_2 \quad l_2m_1)}{(l_1m \quad lm_1)(l_2m \quad lm_2)}\right|$$

$$= \frac{1}{2} \frac{n^2\sqrt{(l_1m_2 \quad l_2m_1)^2 \quad 4l_1l_2m_1m_2}}{|l_1l_2m^2 \quad lm(l_1m_2 \quad l_2m_1) \quad m_1m_2l^2|}$$

$$= \frac{1}{2} \frac{n^2\sqrt{4h^2 \quad 4ab}}{|am^2 \quad 2hlm \quad bl^2|}$$

(Since $l_1l_2 = a$, $m_1m_2 = b$ and $l_1m_2 + l_2m_1 = 2h$)

$$= \frac{n^2\sqrt{h^2 \quad ab}}{|am^2 \quad 2hlm \quad bl^2|}.$$

20. Show that the lines joining the origin to the points of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the line $x - y - \sqrt{2} = 0$ are mutually perpendicular.

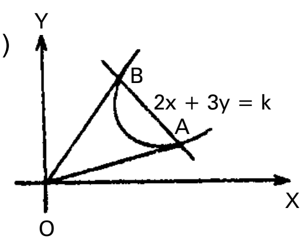
Sol. Equation of the curve is

$x^2 - xy + y^2 + 3x + 3y - 2 = 0$ —— (1)

Equation of AB is $x - y - \sqrt{2} = 0$

$x - y = \sqrt{2}$

$\frac{x - y}{\sqrt{2}} = 1$ —— (2)



Homogenising, (1) with the help of (2) combined equation of OA, OB is

$$x^2 - xy + y^2 + 3x.1 + 3y.1 - 2.1^2 = 0$$

$$x^2 - xy + y^2 + 3(x + y)\frac{x - y}{\sqrt{2}} - 2\frac{(x - y)^2}{2} = 0$$

$$x^2 - xy + y^2 + \frac{3}{\sqrt{2}}(x^2 - y^2) - (x^2 - 2xy + y^2) = 0$$

$$x^2 - xy + y^2 + \frac{3}{\sqrt{2}}x^2 - \frac{3}{\sqrt{2}}y^2 - x^2 + 2xy - y^2 = 0$$

$$\frac{3}{\sqrt{2}}x^2 + xy - \frac{3}{\sqrt{2}}y^2 = 0$$

$$a + b = \frac{3}{\sqrt{2}} - \frac{3}{\sqrt{2}} = 0$$

∴ OA, OB are perpendicular.

21. Find the direction cosines of two lines which are connected by the relations $l + m + n = 0$ and $mn - 2nl - 2lm = 0$.

Sol. Given $l + m + n = 0$ ——— (1)

$mn - 2nl - 2lm = 0$ ——— (2)

From (1), $l = -(m + n)$

Substituting in (2),

$$mn + 2n(m + n) + 2m(m + n) = 0$$

$$mn + 2mn + 2n^2 + 2m^2 + 2mn = 0$$

$$2m^2 + 5mn + 2n^2 = 0$$

$$(2m + n)(m + 2n) = 0$$

$$2m = -n \text{ or } m = -2n$$

Case (i) : $2m_1 = -n_1$

From $l_1 = -m_1 - n_1$

$= -m_1 + 2m_1 = m_1$

$$\frac{l_1}{1} = \frac{m_1}{1} = \frac{x_1}{-2}$$

D.Rs of the first line are 1, 1, –2

D.Cs of this line are $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}$

Case (ii) : $m_2 = -2n_2$

From (1) $l_2 = -m_2 - n_2 = +2n_2 - n_2 = n_2$

$$\frac{l_2}{1} = \frac{m_2}{-2} = \frac{n_2}{1}$$

d.cs of the second line are 1, –2, 1

d.cs of this line are $\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

22. If $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$ $0 < |x| < 1$, then find $\frac{dy}{dx}$.

Sol. Put $x^2 = \cos 2\theta$

$$y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right) = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}+\theta\right)$$

$$= \frac{\pi}{4} + \theta$$

$$= \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{(-1)}{\sqrt{1-x^4}} \times 2x = \frac{-x}{\sqrt{1-x^4}}$$

23. Find the lengths of subtangent, subnormal at a point t on the curve y = a (sin t – t cos t), x = a(cos t + t sin t).

Sol. Equations of the curve are $x = a(\cos t + t \sin t)$

$$x = a(\cos t + t\sin t)$$

$$\frac{dx}{dt} = a(-\sin t + t\cos t) = at\cos t$$

$$y = a(\sin t - t\cos t)$$

$$\frac{dy}{dt} = a(\cos t - \cos t + t\sin t)$$

$$= at\sin t$$

$$\frac{dy}{dt} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{at\sin t}{at\cos t} = \tan t$$

Length of the sub-tangent $= \dfrac{y_1}{f'(x_1)} = \dfrac{a(\sin t - t\cos t)}{\tan t}$

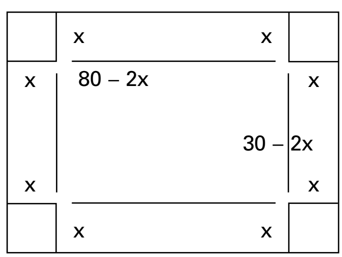
$= |a\cot t(\sin t - t\cos t)|$

Length of the sub-normal $= |y_1 . f'(x_1)|$

$= |a(\sin t - t\cos t)\tan t|$

$= |a\tan t(\sin t - t\cos t|$

24. From a rectangular sheet of dimensions 30 cm × 80 cm four equal squares of side x cms are removed at the corners and the sides are taken turned up so as to form an open rectangular box. Find the value of x, so that the volume of the box is the greatest.



Sol. Length of the box $= 80 - 2x = l$

Breadth of the box $= 30 - 2x = b$

Height of the box $= x = h$

Volume $= lbh$

$= (80 - 2x)(30 - 2x).\, x$

$= x(2400 - 220x + 4x^2)$

$f(x) = 4x^3 - 220x^2 + 2400x$

$f'(x) = 12x^2 - 440x + 2400$

$= 4[3x^2 - 110x + 600]$

$f'(x) = 0 \Rightarrow 3x^2 - 110x + 600 = 0$

$$x = \frac{110 \pm \sqrt{12100 - 7200}}{6}$$

$$= \frac{110 \pm 70}{6} = \frac{180}{6} \text{ or } \frac{40}{6} = \frac{30}{3} \text{ or } \frac{20}{3}$$

If $x = 30$,

$b = 30 - 2x$

$= 30 - 2(30)$

$= -30 < 0$

$\Rightarrow x \neq 30$

$\therefore x = \frac{20}{3}$

$f''(x) = 24x - 440$

When $x = \frac{20}{3}$, $f''(x) = 24.\frac{20}{3} - 440$

$= 160 - 440$

$= -280 < 0$

f(x) is maximum when $x = \frac{20}{3}$

Volume of the box is maximum when $x = \frac{20}{3}$ cm.

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