SOLUTIONS FOR PRACTICE PAPER - 4

SECTION – A

I.1. Find the area of the triangle formed by the straight line $3x - 4y + 12 = 0$ with co-ordinate axes.

Sol. Given straight line equation is $3x - 4y + 12 = 0$ ——— (1)

$\Rightarrow \quad 3x - 4y = -12$

$$\Rightarrow \quad \frac{3x}{-12} - \frac{4y}{-12} = 1$$

$$\Rightarrow \quad \frac{x}{-4} + \frac{y}{3} = 1$$

X – intercept = –4

Y – intercept = 3

Area of the triangle formed by the straight line (1) with the co-ordinate axes is $\frac{1}{2}$ |(X – intercept) (Y – intercept)|

$$= \frac{1}{2} |(-4)(3)|$$

= 6 sq. units.

2. Find the equation of straight line passing through (–2, 4) and making non-zero intercepts whose sum is zero.

Sol. Let X – intercept = a

Y – intercept = b

By hypothasis $a + b = 0$

$\Rightarrow b = -a$

Intercept form $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{a} + \frac{y}{-a} = 1$$

$$\Rightarrow x - y = a$$

But this straight line passing through (–2, 4).

$$\therefore\ -2 - 4 = a$$

$$\Rightarrow a = -6$$

∴ Required straight line equation is

$$x - y = -6$$

$$\Rightarrow x - y + 6 = 0$$

3. Find the angle between the planes 2x−y+z=6, x+y+2z = 7.

Sol. Given plane equations are $2x - y + z - 6 = 0$ —— (1)

$x + y + 2z - 7 = 0$ —— (2)

Let 'θ' be the acute angle between the planes (1) & (2)

$$\therefore \cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$= \left|\frac{2(1) + (-1)(1) + 1(2)}{\sqrt{4+1+1}\ \sqrt{1+1+4}}\right|$$

$$= \left|\frac{2 - 1 + 2}{\sqrt{6}\ \sqrt{6}}\right|$$

$$= \frac{3}{6} = \frac{1}{2}$$

$$\therefore \theta = 60^0$$

4. If (3, 2, –1), (4, 1, 1), (6, 2, 5) are three vertices and (4, 2, 2) is the centroid of a tetrahedron, find the fourth vertex.

Sol. Let A = (3, 2, –1)

B = (4, 1, 1)

C = (6, 2, 5)

D = (x, y, z)

Given (4, 2, 2) is the centroid of the tetrahedron ABCD

$\therefore$ $(4, 2,2) = \left(\frac{3+4+6+x}{4}, \frac{2+1+2+y}{4}, \frac{-1+1+5+z}{4}\right)$

$\frac{13+x}{4} = 4$	$\frac{5+y}{4} = 2$	$\frac{5+z}{4} = 2$
$\Rightarrow$ 13 + x = 16	5 + y = 8	5 + z = 8
$\Rightarrow$ x = 3	y = 3	z = 3

$\therefore$ Fourth vertex D = (3, 3, 3)

5. Compute $\underset{x \to 0}{\text{Lt}} \frac{a^x - 1}{b^x - 1}$; (a > 0, b > 0, b ≡ 1).

Sol. $\lim_{x \to 0} \frac{a^x - 1}{b^x - 1} = \lim_{x \to 0} \frac{\frac{a^x - 1}{x}}{\frac{b^x - 1}{x}}$

$$= \frac{\lim_{x \to 0} \frac{a^x - 1}{x}}{\lim_{x \to 0} \frac{b^x - 1}{x}}$$

$$= \frac{\log e^a}{\log e^b}$$

$$= \log_b^a$$

$\therefore$ $\lim_{x \to 0} \frac{a^x - 1}{b^x - 1} = \log_b^a$.

6. Find $\underset{x \to 0^+}{\text{Lt}} \left(\frac{2|x|}{x} + x + 1\right)$.

Sol. $\lim_{x \to 0} \left(\frac{2|x|}{x} + x + 1\right) = \lim_{x \to 0^+} \left(\frac{2x}{x} + x + 1\right)$ since |x| = x, x > 0

$$= \lim_{x \to 0^+} (3 + x)$$

$$= 3$$

$\therefore$ $\lim_{x \to 0} \left(\frac{2|x|}{x} + x + 1\right) = 3$.

7. If $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, find $\frac{dy}{dx}$.

Sol. Given $y = \text{Tan}^{-1}\left(\frac{2x}{1-x^2}\right)$

Put $x = \text{Tan}\theta \Rightarrow \theta = \text{Tan}^{-1}x$

$$y = \text{Tan}^{-1}\left(\frac{2\,\text{Tan}\theta}{1-\text{Tan}^2\theta}\right)$$

$$= \text{Tan}^{-1}(\text{Tan}\,2\theta)$$

$$= 2\theta$$

$$= 2\text{Tan}^{-1}x$$

differentiating w.r.to 'x' on bothsides, we have

$$\frac{dy}{dx} = 2\frac{d}{dx}(\text{Tan}^{-1}x)$$

$$= 2\left(\frac{1}{1+x^2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}.$$

8. If $y = ae^{nx} + be^{-nx}$, then prove that $y^n = n^2y$.

Sol. Given $y = ae^{nx} + be^{-nx}$

$y^1 = ae^{nx} . n + be^{-nx}(-n)$

$= ane^{nx} - bne^{-nx}$

$y^{11} = ane^{nx} . n - bne^{-nx}(-n)$

$= an^2e^{nx} + bn^2e^{-nx}$

$= n^2(ae^{nx} + be^{-nx})$

$= n^2y$

$\therefore y^{11} = n^2y.$

9. If y = f(x) = x² + x, x = 10, Δx = 0.1, find Δy, dy.

Sol. Given $y = x^2 + x$, $x = 10$ and $\Delta x = 0.1$

$\Delta y = f(x + \Delta x) - f(x)$

$= (x + \Delta x)^2 + (x + \Delta x) - (x^2 + x)$

$= x^2 + 2x\,\Delta x + (\Delta x)^2 + x + \Delta x - x^2 - x$

$= 2x\,\Delta X + (\Delta x)^2 + \Delta x$

$= 2(10)\,(0.1) + (0.1)^2 + 0.1$

$= 2 + 0.01 + 0.1$

$= 2.11$

$dy = f^1(x)\,\Delta x$

$= (2x + 1)\,\Delta x$

$= \{2.(10) + 1\}\,(0.1)$

$= (21)\,(0.1)$

$= 2.10.$

10. Verify Rolle's theorem of the function

log (x² + 2) – log 3 on [–1, 1]

Sol. Let $f(x) = \log(x^2 + 2) - \log 3$

Clearly f is continous on [–1, 1] and

f is derivable on (–1, 1)

Also, $f(-1) = \log(1 + 2) - \log 3 = \log 3 - \log 3 = 0$

$f(1) = \log(1 + 2) - \log 3 = \log 3 - \log 3 = 0$

$\therefore f(-1) = f(1)$

f satisfies all the conditions of Rolle's theorem.

$\therefore$ There exists $C \in (-1, 1)$ such that $f^1(c) = 0$

But $f(x) = \log(x^2 + 2) - \log 3$

$$f^1(x) = \frac{1}{x^2+2}\,.\,2x - 0$$

$$f^1(x) = \frac{2x}{x^2+2}$$

$$f^1(c) = \frac{2c}{c^2+2}$$

$$0 = \frac{2c}{c^2+2}$$

$2c = 0$

$\Rightarrow c = 0 \in (-1, 1)$

Hence Rolle's theorem is verified.

SECTION – B

II.11. Find the equation of the locus of P, if A = (2, 3), B = (2, –3) and PA + PB = 8.

Sol. Given $A = (2, 3)$

$B = (2, -3)$

Let P(x, y) be any point on the locus.

Given geometric condition is PA + PB = 8

$\Rightarrow PA = 8 - PB$

$\Rightarrow PA^2 = (8 - PB)^2$

$\Rightarrow PA^2 = 64 + PB^2 - 16PB$

$\Rightarrow (x-2)^2 + (y-3)^2 = 64 + (x-2)^2 + (y+3)^2 - 16\sqrt{(x-2)^2 + (y+3)^2}$

$\Rightarrow y^2 - 6y + 9 - y^2 - 6y - 9 - 64 = -16\sqrt{x^2 - 4x + 4 + y^2 + 6y + 9}$

$\Rightarrow (-12y - 64) = -16\sqrt{x^2 + y^2 - 4x + 6y + 13}$

$\Rightarrow (3y + 16) = 4\sqrt{x^2 + y^2 - 4x + 6y + 13}$

S.O.B.S

$\Rightarrow (3y + 16)^2 = 16 (x^2 + y^2 - 4x + 6y + 13)$

$\Rightarrow 9y^2 + 256 + 96y = 16x^2 + 16y^2 - 64x + 96y + 208$

$\Rightarrow 16x^2 + 7y^2 - 64x - 48 = 0$

$\therefore$ The locus of P is $16x^2 + 7y^2 - 64x - 48 = 0$

12. When the axes are rotated through an angle $\pi/6$, find the transformed equation of $x^2 + 2\sqrt{3}\,xy - y^2 = 2a^2$.

Sol. Since $\theta = \dfrac{\pi}{6}$, $x = X\cos\alpha - Y\sin\alpha$

$$x = X\cos\frac{\pi}{6} - Y\sin\frac{\pi}{6}$$

$$= X.\frac{\sqrt{3}}{2} - Y.\frac{1}{2} = \frac{\sqrt{3}X - Y}{2}$$

$$y = X\sin\alpha + Y\cos\alpha = X.\sin\frac{\pi}{6} + Y\cos\frac{\pi}{6}$$

$$= X.\frac{1}{2} + Y.\frac{\sqrt{3}}{2} = \frac{X + \sqrt{3}Y}{2}$$

Transformed equation is $f\left(\dfrac{\sqrt{3}X - Y}{2}, \dfrac{X + \sqrt{3}Y}{2}\right) = 0$

$$\Rightarrow \left(\frac{\sqrt{3}X - Y}{2}\right)^2 + 2\sqrt{3}\left(\frac{\sqrt{3}X - Y}{2}\right)\left(\frac{X + \sqrt{3}Y}{2}\right) - \left(\frac{X + \sqrt{3}Y}{2}\right)^2 = 2a^2$$

$$\Rightarrow \frac{3x^2 - 2\sqrt{3}XY + Y^2}{4} + \frac{2\sqrt{3}\,[\sqrt{3}X^2 - XY + 3XY - \sqrt{3}Y^2]}{4} - \frac{X^2 + 3Y^2 + 2\sqrt{3}XY}{4} = 2a^2$$

$$\Rightarrow 3X^2 - 2\sqrt{3}\,XY + Y^2 + 2\sqrt{3}\,[\sqrt{3}X^2 + 2XY - \sqrt{3}Y^2] - (x^2 + 3Y^2 + 2\sqrt{3}XY) = 8a^2$$

$$\Rightarrow 3X^2 - 2\sqrt{3}XY + Y^2 + 6X^2 + 4\sqrt{3}XY - 6Y^2 - X^2 - 3Y^2 - 2\sqrt{3}\,XY = 8a^2$$

$$\Rightarrow 8X^2 - 8Y^2 = 8a^2 \Rightarrow X^2 - Y^2 = a^2$$

13. Find the points on the line 3x – 4y – 1 = 0 which are at a distance of 5 units from the point (3, 2).

Sol. Given lines $x - 7y - 22 = 0$ —— (1)

$3x + 4y + 9 = 0$ —— (2)

$7x + y - 54 = 0$ —— (3)

Let 'A' be the angle between (1), (2)

$$\cos A = \frac{|a_1a_2 + b_1b_2|}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}$$

$$= \frac{|3 - 28|}{\sqrt{1 + 49}\sqrt{9 + 16}} = \frac{25}{5\sqrt{2}.5}$$

$$= \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$A = 45°$

Let 'B' be the angle between (2), (3)

$$\cos B = \frac{21 + 4}{\sqrt{9 + 16}\sqrt{49 + 1}} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$B = 45°$

Let 'C' be the angle between (3), (1)

$$\cos C = \frac{7 - 7}{\sqrt{1 + 49}\sqrt{49 + 1}} = 0 = \cos 90°$$

$C = 90°$

Since $\angle A = \angle B = 45°$

$\angle C = 90°$

∴ Given lines form a right angled isosceles triangle.

14. Verify the continuity of f(x) given by

$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x^2 - 2x - 3} & \text{if } 0 < x < 5 \text{ and } x \neq 3 \\ 1.5 & \text{if } x = 3 \end{cases} \quad \text{at the point 3.}$$

Sol. Given $f(x) = \begin{cases} \dfrac{x^2 - 9}{x^2 - 2x - 3} & \text{if } 0 < x < 5 \text{ and } x \neq 3 \\ 1.5 & \text{if } x = 3 \end{cases}$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^2 - 9}{x^2 - 2x - 3}$$

$$= \lim_{x \to 3} \frac{(x+3)(x-3)}{(x-3)(x+1)}$$

$$= \lim_{x \to 3} \frac{x+3}{x+1}$$

$$= \frac{3+3}{3+1}$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

$$= 1.5$$

$$= f(3)$$

$\therefore \lim_{x \to 3} f(x) = f(3)$

$\therefore$ f is continuous at x = 3.

15. Find the derivative of x sin x from the first principle.

Sol. Let f(x) = x sin x

By first principle

$$f^1(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)\sin(x+h) - x\sin x}{h}$$

$$= \lim_{h \to 0} \frac{x\{\sin(x+h) - \sin x\} + h\sin(x+h)}{h}$$

$$= \lim_{h \to 0} \frac{x \,.\, 2\,\text{Cos}\left(\frac{x+h+x}{2}\right)\text{Sin}\left(\frac{x+h-x}{2}\right) + h\,\text{Sin}(x+h)}{h}$$

$$= \lim_{h \to 0} \frac{2x\,\text{Cos}\left(\frac{2x+h}{2}\right)\text{Sin}\left(\frac{h}{2}\right)}{h} + \lim_{h \to 0} \frac{h\sin(x+h)}{h}$$

$$= 2x\cos\left(\frac{2x+0}{2}\right).\frac{1}{2} + \sin(x+0)$$

$$= x\cos x + \sin x$$

$\therefore\ f^1(x) = x\cos x + \sin x$

16. The volume of a cube is increasing at the rate of 8 cm^3 / sec. How fast is the surface area increasing when the length of an edge is 12 cm ?

Sol. Let x, v, s be length of the edge, volume and surface area of the cube respectively.

Given x = 12 cm.

$$\frac{dv}{dt} = 8\ cm^3/sec.$$

We know $v = x^3$

$$\frac{dv}{dt} = 3x^2\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{3x^2}\frac{dv}{dt}$$

$$= \frac{1}{3.12.12} . 8 = \frac{1}{54}$$

$s = 6x^2$

$\frac{ds}{dt} = 6.2x \cdot \frac{dx}{dt}$

$= 6.2.12. \frac{1}{54} = \frac{8}{3}$ cm²/sec

$\therefore$ Surface area increasing at the rate of $\frac{8}{3}$ cm^2/sec.

17. A particle is moving in a straight line so that after 't' seconds its distance is 'S' (in cms) from a fixed point on the line is given by $S = f(t) = 8t + t^3$. Find i) the velocity at time t = 2 sec ii) the initial velocity iii) acceleration at t = 2 sec.

Sol. Given $S = f(t) = 8t + t^3$

i) $\frac{ds}{dt} = 8 + 3t^2$

$\left(\frac{ds}{dt}\right)_{t=2} = 8 + 3(2)^2$

$= 8 + 12$

$= 20$

$\therefore$ The velocity at time t = 2 is 20 cm / sec

ii) $\left(\frac{ds}{dt}\right)_{t=0} = 8 + 0 = 8$

$\therefore$ Initial velocity = 8cm / sec

iii) $\frac{d^2s}{dt^2} = 0 + 6t$

$\left(\frac{d^2s}{dt^2}\right)_{t=2} = 6(2) = 12$ cm / sec^2

Acceleration at t = 2 is 12 cm / sec^2

SECTION – C

III.18. Find the orthocentre of the triangle formed by the vertices (–2, –1), (6, –1) and (2, 5).

Sol. A(–2, –1) B(6, –1), C(2, 5) are the vertices of ΔABC

Slope of BC = $\frac{5+1}{2-6} = \frac{6}{-4} = -\frac{3}{2}$

AD is perpendicular to BC

Slope of AD = $\frac{2}{3}$

Equation of AD is $y + 1 = \frac{2}{3}(x + 2)$

$2x - 3y + 1 = 0$ —— (1)

Slope of AC = $\frac{5+1}{2+2} = \frac{6}{4} = \frac{3}{2}$

BE is $\perp^{Lr}$ to AC

Slope of BE = $-\frac{2}{3}$

Equation of BE is $y + 1 = -\frac{2}{3}(x - 6)$

$2x + 3y - 9 = 0$ —— (2)

By solving (1), (2)

$$\begin{array}{cccc} 3 & -9 & 2 & 3 \\ -3 & 1 & 2 & -3 \end{array}$$

$$\frac{x}{3-27} = \frac{y}{-18-2} = \frac{1}{-6-6}$$

$$\frac{x}{-24} = \frac{y}{-20} = \frac{1}{-12}$$

$x = \frac{-24}{-12} = 2,\ y = \frac{-20}{-12} = \frac{5}{3}$

$\therefore$ Co-ordinates of the ortho centre O are = $\left[2, \frac{5}{3}\right]$.

19. If the equation $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of parallel straight lines, then show that

i) $h^2 = ab$, ii) $af^2 = bg^2$ and

iii) the distance between the parallel lines $2\sqrt{\dfrac{g^2 - ac}{a(a+b)}}$

Sol. Let the two parallel straight lines represented by S = 0 be

$lx + my + n_1 = 0$ —— (1)

and $lx + my + n_2 = 0.$ —— (2)

Then, $S \equiv \lambda(lx + my + n_1)(lx + my + n_2)$, for some real $\lambda \neq 0$.

From this we have

$$l^2 = \frac{a}{\lambda},\ m^2 = \frac{b}{\lambda},\ n_1 n_2 = \frac{c}{\lambda},\ lm = \frac{h}{\lambda},$$

$$l(n_1 + n_2) = \frac{2g}{\lambda} \text{ and } m(n_1 + n_2) = \frac{2f}{\lambda}.$$

Now (i) $h^2 = \lambda^2 l^2 m^2 = (\lambda l^2)(\lambda m^2) = ab.$

(ii) $4af^2 = (\lambda^2 l^2)(\lambda^2 m^2)(n_1 + n_2)^2$

$= (\lambda m^2)(\lambda^2 l^2)(n_1 + n_2)^2 = b(4g^2)$

so that $af^2 = bg^2$.

(iii) Distance between the parallel lines

$$= \frac{|n_1 - n_2|}{\sqrt{l^2 + m^2}} = \sqrt{\frac{(n_1 + n_2)^2 - 4n_1 n_2}{l^2 + m^2}} = \sqrt{\frac{\lambda^2 (n_1 + n_2)^2 - 4\lambda^2 n_1 n_2}{\lambda^2 (l^2 + m^2)}}$$

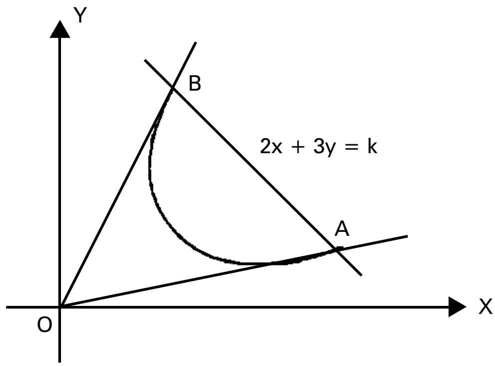
$$= \sqrt{\frac{\frac{4g^2}{l^2} - 4\lambda c}{\lambda(a+b)}} \text{ (or) } \sqrt{\frac{\frac{4f^2}{m^2} - 4\lambda c}{\lambda(a+b)}}$$

$$= 2\sqrt{\frac{g^2 - ac}{a(a+b)}} \text{ (or) } 2\sqrt{\frac{f^2 - bc}{b(a+b)}}.$$

20. Show that the lines joining the origin to the points of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$ are mutually perpendicular.

Sol. Equation of the curve is

$x^2 - xy + y^2 + 3x + 3y - 2 = 0$ —— (1)



Equation of AB is $x - y - \sqrt{2} = 0$

$$x - y = \sqrt{2}$$

$$\frac{x-y}{\sqrt{2}} = 1 \quad \text{—— (2)}$$

Homogenising, (1) with the help of (2) combined equation of OA, OB is

$x^2 - xy + y^2 + 3x.1 + 3y.1 - 2.1^2 = 0$

$x^2 - xy + y^2 + 3(x + y)\frac{x-y}{\sqrt{2}} - 2\frac{(x-y)^2}{2} = 0$

$x^2 - xy + y^2 + \frac{3}{\sqrt{2}}(x^2 - y^2) - (x^2-2xy + y^2) = 0$

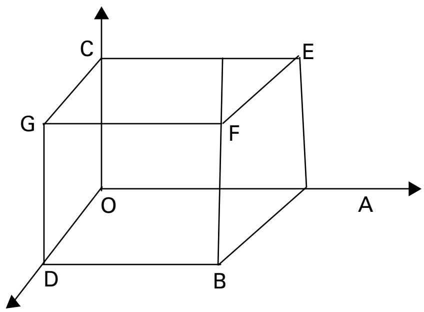
$x^2 - xy + y^2 + \frac{3}{\sqrt{2}}x^2 - \frac{3}{\sqrt{2}}y^2 - x^2 + 2xy - y^2 = 0$

$\frac{3}{\sqrt{2}}x^2 + xy - \frac{3}{\sqrt{2}}y^2 = 0$

$a + b = \frac{3}{\sqrt{2}} - \frac{3}{\sqrt{2}} = 0$

$\therefore$ OA, OB are perpendicular.

21. Find the angle between two diagonals of a cube.



Sol : Let 'O' one of the vertices of the cube taken as origin and the three sides OA, OB, OC are taken as Co-ordinate axes. Let OA = OB = OC = a the four diagonals are $\overrightarrow{OF}$, $\overrightarrow{AG}$, $\overrightarrow{DE}$ and $\overrightarrow{BC}$

The Co-ordinates of the vertices of the cube are

O(0, 0, 0), A(a, 0, 0), B(0, a, 0), C(0, 0, a)

F(a, a,0), D(a, a, 0), E(a, 0, a), G(0, a, a)

D.Rs of OF are (a – 0, a – 0, a – 0) = (a, a, a)

D.Rs of AG are (0 – a, a – 0, a – 0) = (–a, a, a)

If O is the angle between the diagonals OF and AG

then $\cos\theta = \frac{|a(-a) + a.a + a.a|}{\sqrt{a^2 + a^2 + a^2}\ \sqrt{a^2 + a^2 + a^2}}$

$= \frac{a^2}{3a^2} = \frac{1}{3}$

$\theta = \cos^{-1}\left(\frac{1}{3}\right)$

Similarly the angle between any pair of diagonals can be shown to be $\cos^{-1}\left(\frac{1}{3}\right)$.

22. If $y = x\sqrt{a^2+x^2} + a^2 \log\left[x+\sqrt{a^2+x^2}\right]$, then find $\frac{dy}{dx}$.

Sol : Given $y = x\sqrt{a^2+x^2} + a^2 \log(x+\sqrt{a^2+x^2})$

$$\frac{dy}{dx} = x\cdot\frac{1}{2\sqrt{a^2+x^2}}(0+2x) + \sqrt{a^2+x^2} +$$

$$a^2\frac{1}{x+\sqrt{a^2+x^2}}\left[1+\frac{1}{2\sqrt{a^2+x^2}}(0+2x)\right]$$

$$= \frac{2x^2}{2\sqrt{a^2+x^2}} + \sqrt{a^2+x^2} + \frac{a^2}{x+\sqrt{a^2+x^2}}\left[1+\frac{2x}{2\sqrt{a^2+x^2}}\right]$$

$$= \frac{x^2}{\sqrt{a^2+x^2}} + \sqrt{a^2+x^2} + \frac{a^2}{x+\sqrt{a^2+x^2}}\left[\frac{\sqrt{a^2+x^2}+x}{\sqrt{a^2+x^2}}\right]$$

$$= \frac{x^2}{\sqrt{a^2+x^2}} + \sqrt{a^2+x^2} + \frac{a^2}{\sqrt{a^2+x^2}}$$

$$= \frac{x^2+a^2}{\sqrt{a^2+x^2}} + \sqrt{a^2+x^2}$$

$$= \sqrt{a^2+x^2} + \sqrt{a^2+x^2}$$

$$= 2\sqrt{a^2+x^2}$$

$$\therefore \frac{dy}{dx} = 2\sqrt{a^2+x^2}.$$

23. Find the positive integers x and y such that x + y = 60 and xy^3 is maximum.

Sol. Given $x + y = 60 \Rightarrow y = 60 - x$

Let $S = xy^3$

$= x(60-x)^3$

$$\frac{ds}{dx} = x \,.\, 3(60 - x)^2 \,(0 - 1) + (60 - x)^3 \,.\, 1$$

$$= -3x(60 - x)^2 + (60 - x)^3$$

$$= (60 - x)^2 + (-3x + 60 - x)$$

$$= (60 - x)^2 \,(60 - 4x)$$

$$\frac{ds}{dx} = 0 \;\Rightarrow (60 - x)^2 \,(60 - 4x) = 0$$

$$\Rightarrow 60 - 4x = 0$$

$$\Rightarrow 4x = 60$$

$$\Rightarrow x = 15$$

$$\frac{d^2s}{dx^2} = (60 - x)^2 \,(-4) + (60 - 4x)\; 2\,(60 - x)\,(-1)$$

$$= -4\,(60 - x)^2 - 2\,(60 - 4x)\,(60 - x)$$

$$\left(\frac{d^2s}{dx^2}\right)_{x = 15} < 0$$

$\therefore$ S is maximum at $x = 15$

$\therefore$ $y = 60 - 15 = 45$

Hence $x = 15$, $y = 45$.

24. Show that the curves $6x^2 - 5x + 2y = 0$, $4x^2 + 8y^2 = 3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Sol. Given curve equations are

$6x^2 - 5x + 2y = 0$ ———— (1)

$4x^2 + 8y^2 = 3$ ———— (2)

Differentiating (1) w.r.to 'x' on both sides

$$12x - 5 + 2\,\frac{dy}{dx} = 0$$

$$\Rightarrow 2\frac{dy}{dx} = 5 - 12x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(5 - 12x)$$

$$m_1 = \left(\frac{dy}{dx}\right)_{(1/2,\,1/2)} = \frac{1}{2}\left(5 - 12 \cdot \frac{1}{2}\right)$$

$$= \frac{1}{2}(5 - 6)$$

$$= -\frac{1}{2}$$

differentiating (2) w.r.to 'x' on both sides

$$8x + 16y\frac{dy}{dx} = 0$$

$$16y\frac{dy}{dx} = -8x$$

$$\frac{dy}{dx} = \frac{-8x}{16y}$$

$$\frac{dy}{dx} = \frac{-x}{2y}$$

$$m_2 = \left(\frac{dy}{dx}\right)_{\left(\frac{1}{2},\frac{1}{2}\right)} = \frac{-1/2}{2\,(1/2)}$$

$$= -1/2$$

$$\therefore \quad m_1 = m_2$$

$\therefore$ Given curves (1) & (2) touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

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