SOLUTIONS FOR PRACTICE PAPER - 4

SECTION – A

I. 1. If $f : R \rightarrow R$, $g : R \rightarrow R$ are defined by $f(x) = 3x - 1$, $g(x) = x^2 + 1$, then find fog(2).

Sol. Given $f(x) = 3x - 1$ and $g(x) = x^2+1$

$$(fog)(2) = f[g(2)] = f[2^2 + 1] = f(5) = 3(5) - 1 = 14$$

$\therefore (fog)(2) = 14$

2. Find the inverse of the function $f : R \rightarrow R$ defined by $f(x) = ax + b$, $(a \neq 0)$, $a, b \in R$.

Sol. Given $f(x) = ax + b$

Let $f(x) = y$

Then $ax + b = y$

$\Rightarrow ax = y - b$

Since $a \neq 0$

$$\Rightarrow x = \frac{y-b}{a}$$

$$\Rightarrow f^{-1}(y) = \frac{y-b}{a}$$

Hence $f^{-1}(x) = \dfrac{x-b}{a}$

3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and $2X + A = B$, then find X.

Sol. $2x + A = B \Rightarrow 2X = B - A$

$$= \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

4. Find the adjoint of the matrix $\begin{pmatrix} \text{Cos } \alpha & -\text{Sin } \alpha \\ \text{Sin } \alpha & \text{Cos } \alpha \end{pmatrix}$.

Sol. Let $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Adj $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, det $A = \cos^2\alpha + \sin^2\alpha = 1$

$$A^{-1} = \frac{\text{AdjA}}{\text{DetA}}$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

5. If $\bar{a}$ = 2i – j + k and $\bar{b}$ = i – 3j – 5k, then find $|\bar{a} \times \bar{b}|$.

Sol. Given $\bar{a} = 2\bar{i} - \bar{j} + \bar{k}$; $\bar{b} = \bar{i} - 3\bar{j} - 5\bar{k}$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -1 & 1 \\ 1 & -3 & -5 \end{vmatrix}$$

$$= \bar{i}(5+3) - \bar{j}(-10-1) + \bar{k}(-6+1)$$

$$= 8\bar{i} + 11\bar{j} - 5\bar{k}$$

$$|\bar{a} \times \bar{b}| = \sqrt{(8)^2 + (11)^2 + (-5)^2}$$

$$= \sqrt{64 + 121 + 25}$$

$$= \sqrt{210}$$

6. If Sinθ = $-\frac{1}{3}$ and θ does not lie in the 3^{rd} quadrant, find the value of Cos θ.

Sol. Given $\sin\theta = -\frac{1}{3} < 0$

The sine function negative in 3^{rd} and 4^{th} quadrants, but 'θ' does not lie in Q_3 .

$\therefore$ θ lies is Q_4

We know $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \left(\frac{-1}{3}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{1}{9}$$

$$\Rightarrow \cos^2\theta = \frac{8}{9}$$

$$\Rightarrow \cos\theta = \pm\frac{2\sqrt{2}}{3}$$

Since 'θ' lies in Q_4

$$\therefore \cos\theta = \frac{2\sqrt{2}}{3}.$$

7. If $\overline{OA}$ = i + j + k , $\overline{AB}$ = 3i – 2j + k , $\overline{BC}$ = i + 2j – 2k, $\overline{CD}$ = 2i + j + 3k then find the vector $\overline{OD}$.

Sol. $\overline{OA} + \overline{AB} + \overline{BC} + \overline{CD} = \overline{OD}$

$$\Rightarrow \overline{OD} = (\overline{i} + \overline{j} + \overline{k}) + (3\overline{i} - 2\overline{j} + \overline{k}) + (\overline{i} + 2\overline{j} - 2\overline{k}) + (2\overline{i} + \overline{j} + 3\overline{k})$$

$$\Rightarrow \overline{OD} = 7\overline{i} + 2\overline{j} + 3\overline{k}$$

8. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the vertices A, B and C respectively of Δ ABC, then find the vector equation of the median through the vertex A.

Sol. Given $\bar{O}A = \bar{a}$

$\bar{O}B = \bar{b}$

$\bar{O}C = \bar{c}$

Let D be the mid point of Bd

The $\overline{OD} = \dfrac{\bar{b}+\bar{c}}{2}$

$\therefore$ The vector equation of the median through the vertex A is

$$\bar{r} = (1-t)\ \bar{a} + t\left(\frac{\bar{b}+\bar{c}}{2}\right) \quad (t \in R)$$

9. Find a Cosine function whose period is 7.

Sol. $\Rightarrow \dfrac{2\pi}{|a|} = 7$

$\Rightarrow |a| = \dfrac{2\pi}{7}$

$\Rightarrow a = \pm \dfrac{2\pi}{7}$

Required cosine function $= \cos\left(\pm\dfrac{2\pi}{7}\right) x$

$= \cos\left(\dfrac{2\pi}{7}\right) x$

10. If Cosh x = Sec θ, then prove that $\text{Tanh}^2\dfrac{x}{2} = \text{Tan}^2\dfrac{\theta}{2}$.

Sol. We know $\text{Tanh}^2\dfrac{x}{2} = \dfrac{\cosh x - 1}{\cosh x + 1}$

$= \dfrac{\text{Sec}\ \theta - 1}{\text{Sec}\ \theta + 1}$

$$= \frac{1-\cos\theta}{1+\cos\theta} = \frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$= \text{Tan}^2\frac{\theta}{2}$$

$$\therefore \text{Tanh}^2\frac{x}{2} = \text{Tan}^2\frac{\theta}{2}.$$

SECTION – B

II.11. If $A = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{pmatrix}$ **and** $B = \begin{bmatrix} 1 & -2 \\ -3 & 0 \\ 5 & 4 \end{bmatrix}$**, then verify that** $(AB)' = B'.A'$.

Sol. Given $A = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{pmatrix}$

$$B = \begin{pmatrix} 1 & -2 \\ -3 & 0 \\ 5 & 4 \end{pmatrix}$$

$$\therefore A' = \begin{pmatrix} 2 & 1 \\ -1 & 3 \\ 2 & -4 \end{pmatrix}$$

$$B' = \begin{pmatrix} 1 & -3 & 5 \\ -2 & 0 & 4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -3 & 0 \\ 5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2+3+10 & -4+0+8 \\ 1-9-20 & -2+0-16 \end{pmatrix} = \begin{pmatrix} 15 & 4 \\ -28 & -18 \end{pmatrix}$$

$$(AB)' = \begin{pmatrix} 15 & -28 \\ 4 & -18 \end{pmatrix}$$

$$B'.\,A' = \begin{pmatrix} 1 & -3 & 5 \\ -2 & 0 & 4 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 3 \\ 2 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} 2+3+10 & 1-9-20 \\ -4+0+8 & -2+0-16 \end{pmatrix} = \begin{pmatrix} 15 & -28 \\ 4 & -18 \end{pmatrix}$$

$\therefore\ (AB)' = B'\,.\,A'$

12. Find the vector equation of the plane passing through the points 4i – 3j – k, 3i + 7j – 10k and 2i + 5j – 7k and show that the point i + 2j – 3k lies in the plane.

Sol. Let $\bar{a} = 4\bar{i} - 3\bar{j} - \bar{k}$

$\bar{b} = 3\bar{i} + 7\bar{j} - 10\bar{k}$

$\bar{c} = 2\bar{i} + 5\bar{j} - 7\bar{k}$

The vector equation of the plane passing through the points

$\bar{a}, \bar{b}, \bar{c}$ is $\bar{r} = (1 - t - s)\,\bar{a} + t\bar{b} + s\bar{c}$ where $t, s \in r$

$\Rightarrow \bar{r} = (1-t-s)\,(4\bar{i} - 3\bar{j} - \bar{k}) + t(3\bar{i} + 7\bar{j} - 10\bar{k}) + s(2\bar{i} + 5\bar{j} - 7\bar{k})$

Let $\bar{i} + 2\bar{j} - 3\bar{k} = (1-t-s)\,(4\bar{i} - 3\bar{j} - \bar{k}) + t(3\bar{i} + 7\bar{j} - 10\bar{k}) +$

$s(2\bar{i} + 5\bar{j} - 7\bar{k})$

$\Rightarrow 1 = 4 - 4t - 4s + 3t + 2s \Rightarrow t + 2s - 3 = 0 \rightarrow (1)$

$2 = -3 + 3t + 3s + 7t + 5s \Rightarrow 10t + 8s - 5 = 0 \rightarrow (2)$

$3 = -1 + t + s - 10t - 7s \Rightarrow 9t + 6s - 2 = 0 \rightarrow (3)$

Solving (1) & (2)

	t	s	1
2	–3	1	2
6	–2	9	6

$$\frac{t}{-4+18} = \frac{s}{-27+2} = \frac{l}{6-18}$$

$$\frac{t}{14} = \frac{s}{-25} = \frac{1}{-12}$$

$$t = \frac{-14}{12}, s = \frac{25}{12}$$

From (3)

$$9t + 6s - 2 = 9\left(\frac{-14}{12}\right)+6\left(\frac{25}{12}\right)-2$$

$$= \frac{-126+150-24}{12}$$

$$= 0$$

$\Rightarrow t = \frac{-14}{12}, s = \frac{25}{12}$ satisfies equation (3)

$\therefore$ The point $\bar{i}+2\bar{j}-3\bar{k}$ lies in the plane.

13. Show that the points (5, – 1, 1), (7, – 4, 7), (1, – 6, 10) and (–1, –3, 4) are the vertices of a rhombus by vectors.

Sol. Let A = (5,– 1,1)

B = (7,– 4, 7)

C = (1, – 6, 10)

D = (– 1, – 3, 4)

Then the position vectors of A,B,C,D are

$$\overline{OA} = 5\bar{i} - \bar{j} + \bar{k}$$

$$\overline{OB} = 7\bar{i} - 4\bar{j} + 7\bar{k}$$

$$\overline{OC} = \bar{i} - 6\bar{j} + 10\bar{k}$$

$$\overline{OD} = -\bar{i} - 3\bar{j} - 4\bar{k}$$

$$\overline{AB} = \overline{OB} - \overline{OA} = 2\bar{i} - 3\bar{j} + 6\bar{k} \Rightarrow |\overline{AB}| = \sqrt{4+9+36} = 7$$

$$\overline{BC} = \overline{OC} - \overline{OB} = -6\bar{i} - 2\bar{j} + 3\bar{k} \Rightarrow |\overline{BD}| = \sqrt{36+4+9} = 7$$

$\overline{CD} = \overline{OD} - \overline{OC} = -2\bar{i} + 3\bar{j} - 6\bar{k} \Rightarrow |\overline{CD}| = \sqrt{4+9+36} = 7$

$\overline{DA} = \overline{OA} - \overline{OD} = 6\bar{i} + 2\bar{j} - 3\bar{k} \Rightarrow |\overline{DA}| = \sqrt{36+4+9} = 7$

$\overline{AC} = \overline{OC} - \overline{OA}$

$= -4\bar{i} - 5\bar{j} + 9\bar{k} \Rightarrow |\overline{AC}| = \sqrt{16+25+81} = \sqrt{122}$

$\overline{BD} = \overline{OD} - \overline{OB}$

$= -8\bar{i} + \bar{j} - 3\bar{k} \Rightarrow |\overline{BD}| = \sqrt{64+1+9} = \sqrt{74}$

$\overline{AC} . \overline{BD} = (-4)(-8) + (-5)(1) + (9)(-3)$

$= 32 - 5 - 27$

$= 0$

$\therefore$ AB = BC = CD = DA, $AC \neq BD$ and $\overline{AC} \perp \overline{BD}$

$\therefore$ ABCD forms a Rhombus.

14. Prove that $\text{Sin}\frac{\pi}{5}\,\text{Sin}\frac{2\pi}{5}.\text{Sin}\frac{3\pi}{5}.\text{Sin}\frac{4\pi}{5} = \frac{5}{16}$.

Sol. L.H.S $= \text{Sin}\frac{\pi}{5}.\sin\frac{2\pi}{5}.\sin\frac{3\pi}{5}.\sin\frac{4\pi}{5}$

$= \text{Sin } 36^0 . \text{Sin } 72^0 . \text{Sin } 108^0 . \text{Sin } 144^0$

$= \text{Sin } 36^0 . \text{Sin } (90^0 - 18^0) . \text{Sin } (90^0 + 18^0) . \text{Sin } (180^0 - 36^0)$

$= \text{Sin } 36^0 . \text{Cos} 18^0 . \text{Cos } 18^0 . \text{Sin } 36^0$

$= \text{Sin}^2 36^0 . \text{Cos}^2 18^0$

$= \left[\frac{\sqrt{10-2\sqrt{5}}}{4}\right]^2 \left[\frac{\sqrt{10+2\sqrt{5}}}{4}\right]^2$

$= \frac{\left(10-2\sqrt{5}\right)}{16} . \frac{\left(10+2\sqrt{5}\right)^2}{16}$

$= \frac{100-\left(2\sqrt{5}\right)^2}{16.16} = \frac{100-20}{16.16}$

$$= \frac{80}{16.16} = \frac{5}{16}$$

$= R.H.S$

$\therefore$ L.H.S = R.H.S

Hence $\text{Sin}\frac{\pi}{5}.\text{Sin}\frac{2\pi}{5}.\text{Sin}\frac{3\pi}{5}.\text{Sin}\frac{4\pi}{5} = \frac{5}{16}$

15. Solve: 4Sin x. Sin 2x . Sin 4x = Sin 3x.

Sol. Given 4sin x sin 2x sin 4x = sin 3x

2 sinx (2 sin 2x sin 4x) = sin 3x

2 sinx (cos 2x – cos 6x) = sin 3x

2 cos2x sinx – 2 cos 6x sinx = sin 3x

sin 3x – sin x – 2 cos 6x sinx = sin 3x

– sin x – 2cos 6x sin x = 0

sin x + 2cos 6x sin x = 0

sin x (1 + 2cos 6x) = 0

sin x = 0 (or) 1+2 cos 6x = 0

x = nπ   2 cos 6x = –1

$\cos 6x = \frac{-1}{2}$

$6x = 2n\pi \pm \frac{2\pi}{3}$

$x = \frac{n\pi}{3} \pm \frac{\pi}{9}$

Hence the general solution is

$x = n\pi$ (or) $x = \frac{n\pi}{3} \pm \frac{\pi}{9}, n \in z$

16. Prove that $\text{Sin}^{-1}\frac{4}{5} + \text{Sin}^{-1}\frac{7}{25} = \text{Sin}^{-1}\frac{117}{125}$

Sol. Let $\text{Sin}^{-1}\left(\frac{4}{5}\right) = \alpha$ and $\text{Sin}^{-1}\left(\frac{7}{25}\right) = \beta$

$\therefore \cos\alpha = \frac{3}{5}$ and $\cos\beta = \frac{7}{25}$

$\text{Sin}(\alpha+\beta) = \text{Sin}\,\alpha\,\text{Cos}\,\beta + \cos\alpha\,\text{Sin}\,\beta$

$= \left(\frac{4}{5}\right)\left(\frac{24}{25}\right) + \left(\frac{3}{5}\right)\left(\frac{7}{25}\right)$

$= \frac{96}{125} + \frac{21}{125}$

$= \frac{117}{125}$

$\therefore \alpha + \beta = \text{Sin}^{-1}\left(\frac{117}{125}\right)$

Hence $\text{Sin}^{-1}\left(\frac{4}{5}\right) + \text{Sin}^{-1}\left(\frac{7}{25}\right) = \text{Sin}^{-1}\left(\frac{117}{125}\right)$

17. If p_1, p_2, p_3 are the altitudes of ΔABC, then show that

$\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{\text{Cot}\,A + \text{Cot}\,B + \text{Cot}\,C}{\Delta}$.

Sol. Given P_1, P_2, P_3 are the altitudes of ΔABC

$\therefore \Delta = \frac{1}{2}\,aP_1 \Rightarrow P_1 = \frac{2\Delta}{a}$

$\Delta = \frac{1}{2}\,bP_2 \Rightarrow P_2 = \frac{2\Delta}{b}$

$\Delta = \frac{1}{2}\,cP_3 \Rightarrow P_3 = \frac{2\Delta}{c}$

$$\text{L.H.S} = \frac{1}{P_1^2}+\frac{1}{P_2^2}+\frac{1}{P_3^2}$$

$$= \frac{a^2}{4\Delta^2}+\frac{b^2}{4\Delta^2}+\frac{c^2}{4\Delta^2}$$

$$= \frac{a^2+b^2+c^2}{4\Delta^2} \quad \rightarrow (1)$$

$$\text{R.H.S} = \frac{\cot A+\cot B+\cot C}{\Delta}$$

$$= \frac{1}{\Delta}\left(\frac{\cos A}{\sin A}+\frac{\cos B}{\sin B}+\frac{\cos C}{\sin C}\right)$$

$$= \frac{1}{\Delta}\left(\frac{b^2+c^2-a^2}{2bc\sin A}+\frac{c^2+a^2-b^2}{2ca\sin B}+\frac{a^2+b^2-c^2}{2ab\sin C}\right)$$

$$= \frac{1}{\Delta}\left(\frac{b^2+c^2-a^2}{4\Delta}+\frac{c^2+a^2-b^2}{4\Delta}+\frac{a^2+b^2-c^2}{4\Delta}\right)$$

$$= \frac{1}{4\Delta^2}(b^2+c^2-a^2+c^2+a^2-b^2+a^2+b^2-c^2)$$

$$= \frac{a^2+b^2+c^2}{4\Delta^2} \quad \longrightarrow (2)$$

From (1) & (2)

L.H.S = R.H.S

Hence $\frac{1}{p_1^2}+\frac{1}{p_2^2}+\frac{1}{p_3^2}=\frac{\cot A+\cot B+\cot C}{\Delta}$

SECTION – C

III. 18. Let f : $A \to B$, I_A and I_B be identify functions on A and B respectively. Then prove that $foI_A = f = I_Bof$.

Sol. Given f : $A \to B$ be a function

Also $I_A : A \to A$ and $I_B : B \to B$ be identity functions

$I_A : A \to A$, $f : A \to B \Rightarrow foI_A : A \to B$

$\therefore foI_A$ and f are defined on same domain A.

Let $a \in A$

Then $(foI_A)(a) = f[I_A(a)]$

$= f(a)$

$\therefore foI_A = f \to (1)$

$f : A \to B$, $I_B : B \to B \Rightarrow I_BoF : A \to B$

$\therefore I_Bof$ and f are defined on the same domain A.

Let $a \in A$

Then $(I_Bof)(a) = I_B[f(a)]$

$= f(a)$

$\therefore I_Bof = f \to (2)$

From (1) and (2)

$foI_A = f = I_Bof$

19. By mathematical induction, $\perp n \in N$, prove that 2.3+3.4+4.5+... upto n terms = $\frac{n(n^2+6n+11)}{3}$.

Sol. The n^{th} term in the given series is (n+1) (n+2)

Let p(n) be the statement :

$$= \frac{n(n^2+6n+11)}{3}$$

and let S(n) be the sum on the left hand side.

Since $S(1) = 2.3 = \frac{(1)(1+6+11)}{3} = 6$

$\therefore$ The statement is true for n = 1

Assume that the statement p(n) is true for n = k

i.e., $S(k) = 2.3+3.4+...+(k+1)(k+2)$

$$= \frac{k(k^2+6k+11)}{3}$$

We show that the statement is true for n=k+1

i.e., We show that S(k+1)

$$= (k+1)\left[\frac{(k+1)^2+6(k+1)+11}{3}\right]$$

We observe that

$S(k+1) = 2.3 + 3.4 + 4.5 + ...+ (k+1)$

$(k+2) + (k+2)(k+3) = S(k)+(k+2)(k+3)$

$$= \frac{k(k^2+6k+11)}{3} + (k+2)(k+3)$$

$$= \frac{k^3+6k^2+11k+3(k^2+5k+6)}{3}$$

$$= \frac{1}{3}[k^3 + 9k^2 + 26k + 18]$$

$$= \frac{1}{3}(k+1)(k^2+8k+18)$$

By synthetic division

k = –1				
	1	9	26	18
	0	–1	–8	–18
	1	8	18	0

$\therefore$ S(k+1) = $\frac{1}{3}$(k + 1) [(k^2 + 2k + 1) + 6(k + 1) + 11]

$$= \frac{1}{3}(k+1)[(k+1)^2+6(k+1)+11]$$

$\therefore$ The statement holds for n = k+1

$\therefore$ By the principle of mathematical induction.

p(n) is true for all $n \in N$

i.e., 2.3 + 3.4 + 4.5 + ...+ (n + 1) (n + 2)

$$= \frac{n(n^2+6n+11)}{3}$$

20. Show that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$ **= (a – 1)^3.**

Sol. $\begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$ $\begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

= (a – 1)^2 [0 (6 – 3) – 0 [3 (a + 1) –3) + 1(a + 1 – 2)]

= (a –1)^2 (a – 1)

= (a – 1)^3 =R.H.S

21. Solve : 3x + 4y + 5z = 18, 2x – y + 8z =13 and 5x – 2y + 7z = 20 by using the matrix inversion method.

Sol. Let $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 18 \\ 13 \\ 20 \end{bmatrix}$

Given equations can be written as AX = B

By matrix inversion method $X = A^{-1}B$ is the solution

$\det A = 3(-7+16) - 4(14-40) + 5(-4+5)$

$= 27 + 104 + 5$

$= 136$

The cofactors of elements of A are

$A_{11} = +(-7+16) = 9,$

$A_{12} = -(-14-40) = 26,$

$A_{13} = +(-4+5) = 1,$

$A_{21} = -(28+10) = -38,$

$A_{22} = +(21-25) = -4.$

$A_{23} = -(-6-20) = 26,$

$A_{31} = +(32+5) = 37,$

$A_{32} = -(24-10) = -14,$

$A_{33} = (-3-8) = -11.$

$$\therefore \text{Adj}A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}A}{\det A} = \frac{1}{136}\begin{bmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{136}\begin{bmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{bmatrix}\begin{bmatrix} 18 \\ 13 \\ 20 \end{bmatrix}$$

$$= \frac{1}{136}\begin{bmatrix} 162 & -494 & +740 \\ 468 & -52 & -280 \\ 18 & +338 & -220 \end{bmatrix}$$

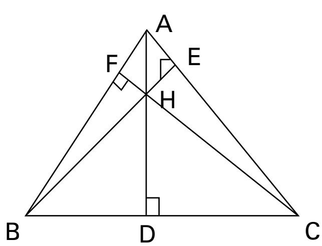
$$= \frac{1}{136}\begin{bmatrix} 408 \\ 136 \\ 136 \end{bmatrix} = \begin{bmatrix} 18 \\ 13 \\ 20 \end{bmatrix}$$

$\therefore$ x = 3, y = 1, z = 1 is the solution.

22. By the vector method, prove that in any triangle, the altitudes are concurrent.

Sol. i) In the given triangle ABC, let the altitudes from A,B drawn to BC, CA respectively intersect them at D,E. Assume that AD and BE intersect at O, join CO and produce it to meet AB at F. With reference to O, let the position vectors of A,B and C be a,b and c respectively.

From Fig. we have



$\overline{BC} = \overline{BO} + \overline{OC} = -\bar{b} + \bar{c}$,

$\overline{CA} = \overline{CO} + \overline{OA} = -\bar{c} + \bar{a}$ and

$\overline{AB} = \overline{AO} + \overline{OB} = -\bar{a} + \bar{b}$

Since $AD \perp BC, \bar{a}.(\bar{c}-\bar{b}) = 0$.

Hence $\bar{a}.\bar{c}-\bar{a}.\bar{b} = 0 \Rightarrow \bar{a}.\bar{c}-\bar{a}.\bar{b}$ ------- (1)

Also, since $\perp AC, \bar{b}.(\bar{a}-\bar{c}) = 0$

Hence $\bar{b}.\,\bar{a}-\bar{b}.\bar{c} = 0 \Rightarrow \bar{b}.\,\bar{a}-\bar{b}.\bar{c}$ ------- (2)

From eq's (1) and (2), We have $\bar{a}.\bar{c} = \bar{b}.\bar{c}$. Hence $\bar{c}.(\bar{b}-\bar{a}) = 0$

$\Rightarrow CF \perp AB$. Hence the altitudes in a triangle are concurrent.

23. If A+B+C= $\frac{3\pi}{2}$ prove that

Cos 2A + Cos 2B + Cos 2C = 1 – 4 SinA. Sin B. Sin C.

Sol. A+B+C = 3π/2 ---------(1)

L.H.S = cos 2A + cos 2B + cos 2C

$= 2\cos(A+B).\cos(A-B) + 1 - 2\sin^2C$

$= 2\cos(270^0 - C).\cos(A-B) - 2\sin^2C$

$= 1 - 2\sin C\cos(A-B) - 2\sin^2 C$

$= 1 - 2\sin C[\cos(A-B) + \sin C]$

$= 1 - 2\sin C\,[\cos(A-B) + \sin(270^0 - \overline{A+B})\,]$

$= 1 - 2\sin C\,[\cos(A-B) - \cos(A+B)]$

$= 1 - 2\sin C\,[2\sin A\sin B]$

$= 1 - 4\sin A\sin B\sin C$

24. Show that $r + r_3 + r_1 - r_2 = 4R \cos B$ in a triangle ABC.

Sol. $r + r_3 = 4R \sin \frac{C}{2}\left[\sin\frac{A}{2}\sin\frac{B}{2}+\cos\frac{A}{2}\cos\frac{B}{2}\right]$

$$= 4R \sin \frac{C}{2}\cos\left(\frac{A-B}{2}\right) \qquad \text{--------}(1)$$

$$r_1 - r_2 = 4R \cos \frac{C}{2}\left[\sin\frac{A}{2}\sin\frac{B}{2}+\cos\frac{A}{2}\cos\frac{B}{2}\right]$$

$$= 4R \cos\frac{C}{2}\sin\left(\frac{A-B}{2}\right) \qquad \text{--------}(2)$$

From (1) and (2)

$\therefore r + r_3 + r_1 - r_2$

$$= 4R\left[\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)+\cos\frac{C}{2}\sin\left(\frac{A-B}{2}\right)\right]$$

$$= 4R \sin\left(\frac{C}{2}+\frac{A}{2}-\frac{B}{2}\right)$$

$$= 4R \sin\left(90^0-\frac{B}{2}-\frac{B}{2}\right) = 4R \cos B.$$

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