

PRACTICE PAPER - 3

SOLUTIONS

SECTION - A

I. 1. Write the conjugate of $\frac{5i}{7+i}$

$$\begin{aligned} \text{Sol. Let } Z &= \frac{5i}{7+i} \\ &= \frac{5i}{7+i} \cdot \frac{7-i}{7-i} = \frac{35i+5}{49+1} = \frac{5}{50} + i\left(\frac{35}{50}\right) \\ &= \frac{1}{10} + i\left(\frac{7}{10}\right) \\ \therefore \bar{z} &= \frac{1}{10} - i\left(\frac{7}{10}\right) \end{aligned}$$

2. If $z \neq 0$, find $\text{Arg } z + \text{Arg } \bar{z}$.

$$\begin{aligned} \text{Sol. If } z &= x + iy, \text{ then } \text{Arg } (z) = \tan^{-1} \left(\frac{y}{x} \right) \\ \text{and } \bar{z} &= x - iy, \text{ then } \text{Arg } (\bar{z}) = \tan^{-1} \left(\frac{-y}{x} \right) = -\tan^{-1} \left(\frac{y}{x} \right) \\ \therefore \text{Arg } (z) + \text{Arg } (\bar{z}) &= \tan^{-1} \frac{y}{x} + \left(-\tan^{-1} \frac{y}{x} \right) = 0 \end{aligned}$$

3. If $x = \text{cis } \theta$, then find the value of $\left(x^6 + \frac{1}{x^6}\right)$

$$\begin{aligned} \text{Sol. } \therefore x &= \cos \theta + i \sin \theta \\ \Rightarrow x^6 &= (\cos \theta + i \sin \theta)^6 = \cos 6\theta + i \sin 6\theta \\ \Rightarrow \frac{1}{x^6} &= \cos 6\theta - i \sin 6\theta \\ \therefore x^6 + \frac{1}{x^6} &= 2 \cos 6\theta \end{aligned}$$

4. For what values of m , the equation $x^2 - 2(1 + 3m)x + 7(3 + 2m) = 0$ will have equal roots?

Sol. The given equation will have equal roots iff its discriminant is 0.

$$\begin{aligned} \text{Here } \Delta &= [(-2(1 + 3m))]^2 - 4(1)[7 + (3 + 2m)] \\ &= 4(1 + 9m^2 + 6m) - 28(3 + 2m) \\ &= 9m^2 - 8m - 20 \\ &= (m - 2)(9m + 10) \end{aligned}$$

$$\text{Hence } \Delta = 0 \Leftrightarrow m = 2, \frac{-10}{9}$$

5. Find the sum of the squares and the sum of the cubes of the roots of the equation $x^3 - px^2 + qx - r = 0$ in terms of p, q, r .

Sol. Let α, β, γ be the roots of the given equation then

$$\alpha + \beta + \gamma = p, \quad \alpha\beta + \beta\gamma + \gamma\alpha = q,$$

$$\alpha\beta\gamma = r$$

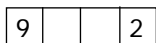
$$\begin{aligned} \text{Sum of the squares of the roots is } &\alpha^2 + \beta^2 + \gamma^2 \\ &= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha) = p^2 - 2q \end{aligned}$$

$$\begin{aligned} \text{Sum of the cubes of the roots is } &\alpha^3 + \beta^3 + \gamma^3 \\ &= (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha) + 3\alpha\beta\gamma \\ &= p(p^2 - 2q - q) + 3r \\ &= p(p^2 - 3q) + 3r \end{aligned}$$

6. How many four digit numbers can be formed using the digits 1, 2, 5, 7, 8, 9? How many of them begin with 9 and end with 2?

Sol. The number of four digit numbers that can be formed using the given digits 1, 2, 5, 7, 8,

9 is ${}^6P_4 = 360$. Now, the first place and last place can be filled with 9 and 2 in one way.



The remaining 2 places can be filled by the remaining 4 digits 1, 5, 7, 8. Therefore, these two places can be filled in 4P_2 ways. Hence, the required number of ways = $1 \cdot {}^4P_2 = 12$.

7. If ${}^{12}P_5 + 5 \cdot {}^{12}P_4 = {}^{13}P_r$, find r.

Sol. We have

$${}^{(n-1)}P_r + r \cdot {}^{(n-1)}P_{(r-1)} = {}^nP_r \text{ and } r \leq n$$

$${}^{12}P_5 + 5 \cdot {}^{12}P_4 = {}^{13}P_5 = {}^{13}P_r \text{ (given)} \Rightarrow r = 5$$

8. Find the 7th term in the expansion of $\left[\frac{4}{x^3} + \frac{x^2}{2} \right]^{14}$.

Sol. The general term in the expansion of $(X + a)^n$ is

$$T_{r+1} = {}^nC_r (X)^{n-r} \cdot a^r$$

$$\text{Put } X = \frac{4}{x^3}, a = \frac{x^2}{2}, n = 14, r = 6$$

$$\begin{aligned} T_7 \text{ in } \left(\frac{4}{x^3} + \frac{x^2}{2} \right)^{14} \text{ is} \\ &= {}^{14}C_6 \left(\frac{4}{x^3} \right)^{14-6} \left(\frac{x^2}{2} \right)^6 \\ &= {}^{14}C_6 \cdot \frac{4^8}{2^6} \cdot \frac{x^{12}}{x^{24}} = {}^{14}C_6 \cdot 4^5 \cdot \frac{1}{x^{12}} \end{aligned}$$

9. Find the mean deviation from the mean of the following discrete data :

6, 7, 10, 12, 13, 4, 12, 6.

Sol. The A.M. of the given data

$$\bar{x} = \frac{6 + 7 + 10 + 12 + 13 + 4 + 12 + 16}{8} = \frac{80}{8} = 10$$

The absolute values of the deviations are

$$|6 - 10|, |7 - 10|, |10 - 10|, |12 - 10|, |13 - 10|, |4 - 10|, |12 - 10|, |16 - 10|$$

$$= 4, 3, 0, 2, 3, 6, 2, 6$$

$$\therefore \text{The mean deviation from the mean} = \frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|$$

$$= \frac{1}{8}(4 + 3 + 0 + 2 + 3 + 6 + 2 + 6)$$

$$= \frac{26}{8} = 3.25$$

10. Let X be a random variable such that $P(X = -2) = P(X = -1) = P(X = 2) = P(X = 1) = \frac{1}{6}$ and $P(X = 0) = \frac{1}{3}$. Then find the mean and variance of X .

Sol. Mean = $(-2) \frac{1}{6} + (-1) \frac{1}{6} + 2 \left(\frac{1}{6}\right) + (1) \left(\frac{1}{6}\right) + 0 \cdot \left(\frac{1}{3}\right)$

$$= -\frac{2}{6} - \frac{1}{6} + \frac{2}{6} + \frac{1}{6} + 0$$

$$\mu = 0$$

Variance (σ^2) = $(-2)^2 \left(\frac{1}{6}\right) + (-1)^2 \left(\frac{1}{6}\right) + 0^2 \left(\frac{1}{3}\right) + 2^2 \left(\frac{1}{6}\right) + 1^2 \left(\frac{1}{6}\right)$

$$= \frac{4}{6} + \frac{1}{6} + 0 + \frac{4}{6} + \frac{1}{6}$$

$$= \frac{10}{6} = \frac{5}{3}$$

SECTION - B

II.11. If $z = 3 - 5i$, then show that $z^3 - 10z^2 + 58z - 136 = 0$

Sol. $z = 3 - 5i$ (1)

$$\Rightarrow z^2 = (3 - 5i)^2 = (3)^2 - 2(3)(5i) + (5i)^2$$

$$\begin{aligned}
 &= 9 - 30i + 25i^2 \\
 &= 9 - 30i + 25(-1) \\
 &= -16 - 30i
 \end{aligned}$$

$$\therefore z^2 = -16 - 30i \quad \dots\dots\dots (2)$$

$$\begin{aligned}
 \text{Now } z^3 &= z^2 \cdot z = (-16 - 30i)(3 - 5i) \\
 &= -48 - 90i + 80i + 150i^2 \\
 &= -48 - 10i + 150(-1) \\
 &= -198 - 10i
 \end{aligned}$$

$$\therefore z^3 = -198 - 10i \quad \dots\dots\dots (3)$$

$$\begin{aligned}
 \text{Now } z^3 - 10z^2 + 58z - 136 & \\
 &= (-198 - 10i) - 10(-16 - 30i) + 58(3 - 5i) - 136 \\
 &= -198 - 10i + 160 + 300i + 174 - 290i - 136 \\
 &= (-198 + 160 + 174 - 136) + i(-10 + 300 - 290) \\
 &= 0 + i(0) = 0
 \end{aligned}$$

$$\therefore z^3 - 10z^2 + 58z - 136 = 0$$

12. Solve $\sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = \frac{5}{2}$, when $x \neq 0$ and $x \neq 3$

Sol. Put $a = \sqrt{\frac{x}{x-3}}$

$$a + \frac{1}{a} = \frac{5}{2}$$

$$\Rightarrow \frac{a^2 + 1}{a} = \frac{5}{2}$$

$$\Rightarrow 2a^2 + 2 = 5a \Rightarrow 2a^2 - 5a + 2 = 0$$

$$\Rightarrow (2a - 1)(a - 2) = 0$$

$$\Rightarrow 2a - 1 = 0 \text{ or } a - 2 = 0$$

$$a = \frac{1}{2} \text{ or } 2$$

Case (i) : $a = 2$

$$\sqrt{\frac{x}{x-3}} = 2 \Rightarrow \frac{x}{x-3} = 4$$

$$\Rightarrow x = 4x - 12$$

$$\Rightarrow 3x = 12 \Rightarrow x = 4$$

Case (ii) : $a = \frac{1}{2}$

$$\sqrt{\frac{x}{x-3}} = \frac{1}{2} \Rightarrow \frac{x}{x-3} = \frac{1}{4} \Rightarrow 4x = x - 3$$

$$3x = -3 \Rightarrow x = -1$$

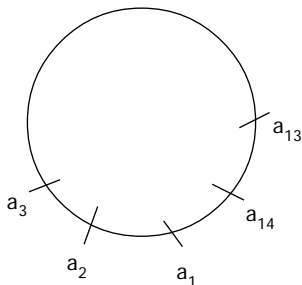
∴ The roots are -1, 4.

- 13.** 14 persons are seated at a round table. Find the number of ways of selecting two persons out of them who are not seated adjacent to each other.

Sol. The seating arrangement of given 14 persons at the round table as shown below.

Number of ways of selecting 2 persons out of 14 persons = ${}^{14}C_2$

$$= \frac{14 \times 13}{1 \times 2} = 91.$$



In the above arrangement two persons sitting adjacent to each other can be selected in 14 ways. (they are $a_1a_2, a_2a_3, \dots, a_{13}a_{14}, a_{14}a_1$)

∴ The required no. of ways = $91 - 14 = 77$

- 14.** In how many ways 9 mathematics papers can be arranged so that the best and the worst i) may come together ii) may not come together ?

Sol. i) If the best and worst papers are treated as one unit, then we have $9 - 2 + 1 = 7 + 1 = 8$ papers Now these can be arranged in

$(7 + 1)!$ ways and the best and worst papers between themselves can be permuted in $2!$ ways. Therefore the number of arrangements in which best and worst papers come together is $8!2!$.

- ii) Total number of ways of arranging 9 mathematics papers is $9!$.
The best and worst papers come together in $8!2!$ ways. Therefore the number of ways they may not come together is $9! - 8!2! = 8!(9 - 2) = 8! \times 7$

15. Resolve $\frac{x^2 + 5x + 7}{(x - 3)^2}$ into partial fractions.

Sol. Let $x - 3 = y \Rightarrow x = y + 3$

$$\begin{aligned} \frac{x^2 + 5x + 7}{(x - 3)^3} &= \frac{(y + 3)^2 + 5(y + 3) + 7}{y^3} \\ &= \frac{y^2 + 6y + 9 + 5y + 15 + 7}{y^3} \\ &= \frac{y^2 + 11y + 31}{y^3} = \frac{1}{y} + \frac{11}{y^2} + \frac{31}{y^3} \\ \therefore \frac{x^2 + 5x + 7}{(x - 3)^3} &= \frac{1}{x - 3} + \frac{11}{(x - 3)^2} + \frac{31}{(x - 3)^3} \end{aligned}$$

16. If A, B, C are three events then show that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

Sol. Write $B \cup C = D$ then $P(A \cup B \cup C) = P(A \cup D)$

$$\begin{aligned} \therefore P(A \cup D) &= P(A) + P(D) - P(A \cap D) \\ &= [P(A) + P(B \cup C) - P(A \cap (B \cup C))] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B) \cup (A \cap C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B) + P(A \cap C) \\ &\quad - P(A \cap B \cap C)] \end{aligned}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$$

17. In a box containing 15 Bulbs, 5 are defective. If 5 bulbs are selected at random from the box, find the probability of the event, that

- i) none of them is defective
- ii) only one of them is defective
- iii) atleast one of them is defective

Sol. Out of 15 bulbs, 5 are defective

$$\text{probability of selecting a defective bulb} = P = \frac{5}{15} = \frac{1}{3}$$

We are selecting 5 bulbs $n(S) = {}^{15}C_5$

i) None of them is defective. All the 5 bulbs must be selected from 10 good bulbs. This can be done in ${}^{10}C_5$ ways.

$$P(A) = \frac{{}^{10}C_5}{{}^{15}C_5} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11} = \frac{12}{143}.$$

ii) Only one of them is defective in 4 good and 1 defective balls.

$$\text{This can be done in } {}^{10}C_4 {}^5C_1 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} \cdot 5 = 210 \times 5 = 1050.$$

$$\text{Probability of selecting one defective} = \frac{1050}{{}^{15}C_5} = (1050)$$

$$\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11} = \frac{50}{143}$$

iii) Probability atleast one of them is defective

$$= 1 - P(A) = 1 - \frac{12}{143} = \frac{131}{143}.$$

SECTION - C

III.18. If α, β are the roots of the equation $x^2 - 2x + 4 = 0$ then for any $n \in \mathbb{N}$, show that

$$\alpha^n + \beta^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$$

Sol. $x^2 - 2x + 4 = 0$

$$x = \frac{2 \pm \sqrt{4-16}}{2} = \frac{2 \pm 2\sqrt{-3}}{2} = 1 \pm i\sqrt{3}$$

$$\text{Let } \alpha = 1 + i\sqrt{3} = 2 \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)$$

$$= 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$\alpha^n = (1 + i\sqrt{3})^n = 2^n \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)^n$$

$$= 2^n \left(\cos \frac{n\pi}{3} + i \sin \frac{n\pi}{3} \right) \dots\dots\dots (1)$$

$$\beta = 1 - i\sqrt{3} = 2 \left(\frac{1}{2} - i \frac{\sqrt{3}}{2} \right)$$

$$= 2 \left(\cos \left(-\frac{\pi}{3} \right) + i \sin \left(-\frac{\pi}{3} \right) \right)$$

$$\beta^n = (1 - i\sqrt{3})^n$$

$$= 2^n \left(\cos \left(-\frac{\pi}{3} \right) + i \sin \left(-\frac{\pi}{3} \right) \right)^n$$

$$= 2^n \left(\cos \left(-\frac{n\pi}{3} \right) + i \sin \left(-\frac{n\pi}{3} \right) \right)$$

$$= 2^n \left(\cos \frac{n\pi}{3} + i \sin \frac{n\pi}{3} \right) \dots\dots\dots (2)$$

Adding (1), (2)

$$\begin{aligned} \alpha^n + \beta^n &= 2^n \left(\cos \frac{n\pi}{3} + i \sin \frac{n\pi}{3} \right) + 2^n \left(\cos \frac{n\pi}{3} - i \sin \frac{n\pi}{3} \right) \\ &= 2^n \left(\cos \frac{n\pi}{3} + i \sin \frac{n\pi}{3} + \cos \frac{n\pi}{3} - i \sin \frac{n\pi}{3} \right) \\ &= 2^n \left(2 \cos \frac{n\pi}{3} \right) = 2^{n+1} \cdot \cos \frac{n\pi}{3} \end{aligned}$$

19. Solve : $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots of this equation are in arithmetic progression.

Sol. Let $a - d, a, a + d$ are the roots of the given equation.

Now, sum of the roots

$$\begin{aligned} a - d + a + a + d &= \frac{24}{4} \\ 3a &= 6 \\ a &= 2 \end{aligned}$$

Product of the roots $(a - d) a (a + d) = \frac{-18}{4}$

$$a(a^2 - d^2) = -\frac{9}{2} = 2(4 - d^2) = -\frac{9}{2}$$

$$4(4 - d^2) = -9$$

$$16 - 4d^2 = -9$$

$$4d^2 = 25$$

$$d = \pm \frac{5}{2}$$

\therefore roots are $-\frac{1}{2}, 2$ and $\frac{9}{2}$

20. Prove that

$$\binom{2n}{0}^2 - \binom{2n}{1}^2 + \binom{2n}{2}^2 - \binom{2n}{3}^2 + \dots + \binom{2n}{2n}^2 = (-1)^n 2^n C_n$$

Sol. $(x + 1)^{2n} = \binom{2n}{0} x^{2n} + \binom{2n}{1} x^{2n-1} + \binom{2n}{2} x^{2n-2} + \dots + \binom{2n}{2n} \dots\dots (1)$

$$(1 - x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \dots + {}^{2n}C_{2n} x^{2n} \dots \dots (2)$$

Multiplying equations (1) & (2), we get

$$({}^{2n}C_0 x^{2n} + {}^{2n}C_1 x^{2n-1} + {}^{2n}C_2 x^{2n-2} + \dots + {}^{2n}C_{2n}) \cdot ({}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \dots + {}^{2n}C_{2n} x^{2n})$$

$$= (x + 1)^{2n} (1 - x)^{2n} = [(1 + x) (1 - x)]^{2n}$$

$$= (1 - x^2)^{2n}$$

$$= \sum_{r=0}^{2n} {}^{2n}C_r (-x^2)^r$$

Equating the coefficients of x^{2n}

$$({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 + \dots + ({}^{2n}C_{2n})^2 (-1)^n {}^{2n}C_n$$

$$\therefore ({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 + \dots + ({}^{2n}C_{2n})^2 = (-1)^n {}^{2n}C_n$$

21. Find the sum of the infinite series $\frac{3}{4.8} - \frac{3.5}{4.8.12} + \frac{3.5.7}{4.8.12.16} \dots$

Sol. Let $S = \frac{3}{4.8} - \frac{3.5}{4.8.12} + \frac{3.5.7}{4.8.12.16} \dots$
 $= \frac{1.3}{4.8} - \frac{1.3.5}{4.8.12} + \frac{1.3.5.7}{4.8.12.16} \dots$

Add $1 - \frac{1}{4}$ on both sides,

$$1 - \frac{1}{4} + S = 1 - \frac{1}{4} + \frac{1.3}{4.8} - \frac{1.3.5}{4.8.12} + \dots$$

$$\Rightarrow \frac{3}{4} + S = 1 - \frac{1}{1} \cdot \frac{1}{4} + \frac{1.3}{1.2} \left(\frac{1}{4}\right)^2$$

$$= \frac{1.3.5}{1.2.3} \left(\frac{1}{4}\right)^3 + \dots$$

$$= 1 - \frac{p}{1} \cdot \frac{x}{q} + \frac{(p)(p+q)}{1.2} \left(\frac{x}{q}\right)^2 - \frac{(p)(p+q)(p+2q)}{1.2.3} \left(\frac{x}{q}\right)^3 + \dots$$

Here $p = 1, q = 2, \frac{x}{q} = \frac{1}{4}$

$$\Rightarrow x = \frac{2}{4} = \frac{1}{2}$$

$$\begin{aligned}
 &= (1 + x)^{-p/q} = \left(1 + \frac{1}{2}\right)^{-1/2} \\
 &= \left(\frac{3}{2}\right)^{-1/2} = \sqrt{\frac{2}{3}} \\
 \therefore S &= \sqrt{\frac{2}{3}} - \frac{3}{4}
 \end{aligned}$$

22. An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following data.

	Firm A	Firm B
Number of workers	500	600
Average daily wage (Rs.)	186	175
Variance of distribution of wages	81	100

- Which firm A or B, has greater variability in individual wages?
- Which firm has larger wage bill.

Sol. Given $\sigma_A^2 = 81 \Rightarrow \sigma_A = 9$

$$\sigma_B^2 = 100 \Rightarrow \sigma_B = 10$$

Also $\bar{x}_A = 186$ and $\bar{x}_B = 175$

$$i) \text{ C.V. of firm A} = \frac{\sigma_A}{\bar{x}_A} \times 100 = \frac{9}{186} \times 100 = 4.84$$

$$\text{C.V. of firm B} = \frac{\sigma_B}{\bar{x}_B} \times 100 = \frac{10}{175} \times 100 = 5.71$$

C.V. of firm B > C.V. of firm A.

\therefore Firm B has greater variability in industrial wages.

ii) Total wages paid to the workers in firm A = $500 \times 186 = 93,000$

Total wages paid to the workers in firm B = $600 \times 175 = 1,05,000$

Hence firm B has larger wage bill.

23. If A, B, C are three independent events of an experiment such that, $P(A \cap B^c \cap C^c) = \frac{1}{4}$, $P(A^c \cap B \cap C^c) = \frac{1}{8}$, $P(A^c \cap B^c \cap C^c) = \frac{1}{4}$ then find P(A),P(B) and P(C).

Sol. Since A, B, C are independent events.

$$P(A \cap B^c \cap C^c) = \frac{1}{4}$$

$$\Rightarrow P(A).P(B^c).P(C^c) = \frac{1}{4} \quad \dots\dots\dots (1)$$

$$P(A^c \cap B \cap C^c) = \frac{1}{8}$$

$$\Rightarrow P(A^c).P(B).P(C^c) = \frac{1}{8} \quad \dots\dots\dots (2)$$

$$P(A^c \cap B^c \cap C^c) = \frac{1}{4}$$

$$P(A^c).P(B^c).P(C^c) = \frac{1}{4} \quad \dots\dots\dots (3)$$

$$\frac{(1)}{(3)} \Rightarrow \frac{P(A)}{P(A^c)} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1$$

$$\Rightarrow \frac{P(A)}{1-P(A)} = 1$$

$$\Rightarrow P(A) = 1 - P(A)$$

$$\Rightarrow 2P(A) = 1 \Rightarrow P(A) = \frac{1}{2} \quad \frac{(2)}{(3)} \Rightarrow \frac{P(B)}{P(B^c)} = \frac{\frac{1}{8}}{\frac{1}{4}} \Rightarrow \frac{P(B)}{1-P(B)} = \frac{1}{2}$$

$$\Rightarrow 2P(B) = 1 - P(B)$$

$$\Rightarrow 3P(B) = 1$$

$$\therefore P(B) = \frac{1}{3}$$

$$\text{From (1) } P(A).P(B^c).P(C^c) = \frac{1}{4}$$

$$\Rightarrow \left(\frac{1}{2}\right) \left(1 - \frac{1}{3}\right) P(C^c) = \frac{1}{4} \Rightarrow P(C^c) = \frac{1}{4} \times 2 \times \frac{3}{2} = \frac{3}{4}$$

$$\therefore P(C) = 1 - P(C^c) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\therefore P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$$

- 24.** If the mean and variance of a binomial variable X are 2.4 and 1.44 respectively, Find $P(1 < X \leq 4)$.

Sol. Mean = $np = 2.4$ ----- (1)

Variance = $npq = 1.44$ ----- (2)

Dividing (2) by (1), $\frac{npq}{np} = \frac{1.44}{2.4}$

$$q = 0.6 = \frac{3}{5}$$

$$p = 1 - q = 1 - 0.6 = 0.4 = \frac{2}{5}$$

Substituting in (1), $n(0.4) = 2.4$

$$n = \frac{2.4}{0.4} = 6$$

$$\begin{aligned} P(1 < X \leq 4) &= P(X = 2) + P(X = 3) + P(X = 4) \\ &= {}^6C_2 q^4 \cdot p^2 + {}^6C_3 q^3 \cdot p^3 + {}^6C_4 q^2 \cdot p^4 \\ &= {}^6C_2 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^2 + {}^6C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^3 + {}^6C_4 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^4 \\ &= \frac{6^2}{5^6} (15.9 + 20.6 + 15.4) \\ &= \frac{36}{15625} (135 + 120 + 60) \\ &= \frac{315 \times 36}{15625} = \frac{36 \times 63}{3125} = \frac{2268}{3125} \end{aligned}$$

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