PRACTICE PAPER – 4

SOLUTIONS

SECTION – A

I. 1. If $Z = \dfrac{1+2i}{1-(1-i)^2}$, then find Arg (Z).

Sol. **Given** $Z = \dfrac{1+2i}{1-(1-i)^2}$

$$= \frac{1+2i}{1-(1-2i+i^2)} = \frac{1+2i}{1+2i} = 1$$

$= \cos 0° + i \sin 0°$

$\therefore$ Arg(Z) = 0°

2. Z = x + iy represents a point in the Argand plane. Find the locus of Z such that |Z| = 2.

Sol. Let Z = x + iy. Then |Z| = 2.

If and only if |x + iy| = 2.

If and only if $\sqrt{x^2+y^2} = 2$. If and only if $x^2 + y^2 = 4$

$x^2 + y^2 = 4$ represents a circle with centre at (0, 0) and radius 2!

$\therefore$ The locus of |z| = 2 is the circle $x^2 + y^2 = 4$.

3. Find the value of $(-16)^{1/4}$

Sol. (–16) = 16(–1) = 16(cos π + i sin π)

$= 2^4 (\cos (2k+1)\pi + i \sin (2k+1)\pi],\ k \in z$

$(-16)^{1/4} = (2^4)^{1/4} [\cos (2k+1)\pi + i \sin (2k+1)\pi]^{1/4}$

$= 2 \text{ cis } \dfrac{(2k+1)\pi}{4}, k = 0, 1, 2, 3$

The value of $(-16)^{1/4}$ is

$2 \text{ cis } \dfrac{(2k+1)\pi}{4}, k = 0, 1, 2, 3$

4. If α, β are the roots of the equation $ax^2 + bx + c = 0$. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ in terms of a, b, c.

Sol. α, β are the roots of the equation $ax^2 + bx + c = 0$

$$\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(-\frac{b^2}{a}\right) - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$= \frac{b^2 - 2ac}{c^2}$$

5. Find the condition that $x^3 - px^2 + qx - r = 0$ may have the roots in G.P.

Sol. Let $\frac{a}{R}$, a, aR be the roots of $x^3 - px^2 + qx - r = 0$

Then product of the roots = $\frac{a}{R}.a.\ aR = a^3 = r \Rightarrow a = r^{1/3}$

$\because$ a is a root of $x^3 - px^2 + qx - r = 0$

$\Rightarrow a^3 - pa^2 + qa - r = 0$

But $a = r^{1/3}$

$\Rightarrow (r^{1/3})^3 - p(r^{1/3})^2 + q(r^{1/3}) - r = 0$

$\Rightarrow r - p.\ r^{2/3} + q.\ r^{1/3} - r = 0 \Rightarrow p.\ r^{2/3} = q\ r^{1/3}$

By cubing on both sides

$\Rightarrow p^3r^2 = q^3r$

$\Rightarrow p^3r = q^3$ is the required condition.

6. If 10. $^nC_2$ = 3. $^{n+1}C_3$, find n.

Sol. 10. $^nC_2 = 3.\ ^{n+1}C_3$

$$\Rightarrow 10 \times \frac{n(n-1)}{1.2} = \frac{3(n+1)(n-1)}{1.2.3}$$

$\Rightarrow$ 10 = n + 1

$\therefore$ n = 9.

7. Find the number of ways of forming a committee of 5 members from 6 men and 3 ladies.

Sol. Total number of persons = 6 + 3 = 9

$\therefore$ Number of ways of forming a committee of 5 members from 6 men and 3 ladies.

$$= {}^9C_5 = \frac{9 \times 8 \times 7 \times 6 \times 5}{5 \times 4 \times 3 \times 2 \times 1} = 126$$

8. Prove that $C_0 + 3.C_1 + 3^2.C_2 + \ldots\ldots\ldots + 3\ ^nC_n = 4^n$.

Sol. We have $(1+x)^n = C_0 + C_1.x + C_2.x^2 + \ldots\ldots + C_n.x^n$

Put x = 3, we get

$(1+3)^n = C_0 + C_1.\ 3 + C_2\ .\ 3^2 + \ldots\ldots + C_n.\ 3^n$

$\therefore C_0 + 3.\ C_1 + 3^2\ .\ C_2 + \ldots\ldots + 3^n.\ C_n = 4^n$

9. Find the mean deviation about the mean for the following distribution.

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Sol. Construct the table

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
10	4	40	40	160
30	24	720	20	480

50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	$N=\Sigma f_i=80$	$\Sigma f_i x_i=4000$		1280

$$\bar{x}=\frac{\Sigma f_i x_i}{\Sigma f_i}=\frac{4000}{80}=50$$

$$\text{Mean deviation } = \frac{1}{N}=\sum_{i=1}^{5} f_i\,|x_i-\bar{x}|$$

$$=\frac{1}{80}(1280) = 16$$

10. In a city 10 accidents take place in a span of 50 days. Assuming that the number of accidents follows the poisson distribution, Find the probability that there will be 3 or more accidents in a day.

Sol. Average number of accidents per day

$$\lambda=\frac{10}{50}=\frac{1}{5}=0.2$$

The probability that there win be 3 or more accidents in a day

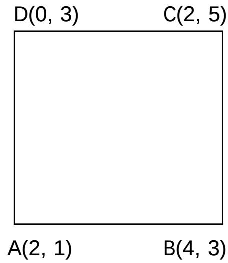
$$P(X\geq 3) = \sum_{K=3}^{\infty} e^{-\lambda}.\frac{\lambda^K}{K!}, \lambda=0.2$$

SECTION – B

II.11. Show that the four points in the Argand plane represented by the complex numbers 2 + i, 4 + 3i, 2 + 5i, 3i are the vertices of a square.

Sol. A(2, 1), B(4, 3), C(2, 5), D(0, 3) represents the given complex number in the Argand plane.

$AB^2 = (2-4)^2 + (1-3)^2 = 4 + 4 = 8$



$BC^2 = (4-2)^2 + (3-5)^2 = 4 + 4 = 8$

$CD^2 = (2-0)^2 + (5-3)^2 = 4 + 4 = 8$

$DA^2 = (0-2)^2 + (3-1)^2 = 4 + 4 = 8$

$\Rightarrow AB^2 = BC^2 = CD^2 = DA^2$

$AB = BC = CD = DA$ .... (1)

$AC^2 = (2-2)^2 + (1-5)^2 = 0 + 16 = 16$

$BD^2 = (4-0)^2 + (3-3)^2 = 16 + 0 = 16$

$AC^2 = BD^2 \Rightarrow AC = BD$ .... (2)

By (1) & (2) $\Rightarrow$ A, B, C, D are the vertices of a square.

12. Suppose that the Quadratic equations $ax^2+bx+c = 0$ and $bx^2+cx+a = 0$ have a common root. Then show that $a^3+b^3+c^3 = 3abc$.

Sol. Let α be the common root of the equations $ax^2+bx+c = 0$ and $bx^2+cx+a = 0$

$\therefore a\alpha^2 + b\alpha + c = 0$

$b\alpha^2 + c\alpha + a = 0$

α^2	α	1	α^2
b	c	a	b
c	a	b	c

$$\frac{\alpha^2}{ab-c^2} = \frac{\alpha}{bc-a^2} = \frac{1}{ac-b^2}$$

$$\alpha^2 = \frac{ab-c^2}{ac-b^2}, \alpha = \frac{bc-a^2}{ac-b^2}$$

$$\therefore \left(\frac{bc-a^2}{ac-b^2}\right)^2 = \frac{ab-c^2}{ac-b^2}$$

$(bc-a^2)^2 = (ab-c^2)(ac-b^2)$

$b^2c^2 + a^4 - 2a^2bc = a^2bc - ab^3 - ac^3 + b^2c^2$

$a^4 + ab^3 + ac^3 = 3a^2bc$

$a(a^3 + b^3 + c^3) = 3a^2bc$

$a^3 + b^3 + c^3 = 3abc \ (\because a \neq 0)$

13. How many numbers can be formed using the digits 1, 2, 3, 4, 3, 2, 1 such that even digits always occupy even places ?

Sol. In the given 7 digits, there are two 1's , two 2's , two 3's and one 4.

The 3 even places can be occupied by the even digits 2, 4, 2, in $\frac{3!}{2!}$. (Even place is shown by E)

	E		E		E	

The remained odd places can be occupied by the odd digits 1, 3, 3, 1 in $\frac{4!}{2!\,2!}$ ways .

$\therefore$ The number of required arrangements $= \frac{3!}{2!} \times \frac{4!}{2! \times 2!} = 3 \times 6 = 18$.

14. Find the number of ways of arranging the letters of the word MISSING so that the two S's are together and the two I's are together.

Sol. In the given word MISSING contains 7 letters in which there are 2I's are a like, 2S's are alike and rest are different.

Treat the 2S's as one unit and 2I's as one unit. Then we have 3 + 1 + 1 = 5 entities.

These can be arranged in 5! ways. The 2S's can be arranged among themselves in $\frac{2!}{2!} = 1$ way and the 2I's can be arranged among themselves in $\frac{2!}{2!} = 1$ way.

$\therefore$ The number of required arrangements $= 5! \times 1 \times 1 = 120$

15. Resolve $\frac{x+4}{(x^2-4)(x+1)}$ into partial Fractions.

Sol. $\frac{x+4}{(x^2-4)(x+1)} = \frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x-2}$

Multiplying with $(x^2 - 4)(x + 1)$

$x + 4 = A(x^2 - 4) + B(x + 1)(x - 2) + C(x + 1)(x + 2)$

$x = -1 \Rightarrow 3 = A(1 - 4) = -3A \Rightarrow A = -1$

$x = -2 \Rightarrow 2 = B(-2 + 1)(-2 - 2) = 4B \Rightarrow B = +\frac{2}{4}=\frac{1}{2}$

$x = 2 \Rightarrow 6 = C(2 + 1)(2 + 2) = 12C \Rightarrow C = \frac{1}{2}$

$\therefore \frac{x+4}{(x^2-4)(x+1)} = -\frac{1}{x+1}+\frac{1}{2(x+2)}+\frac{1}{2(x-2)}$

16. The probability for a contractor to get a road contract is $\frac{2}{3}$ and to get a building contract is $\frac{5}{9}$. The probability to get atleast one contract is $\frac{4}{5}$. Find the probability that he gets both the contracts.

Sol. Suppose A is the event of getting a road contract.

B is the event of getting a building contract

Given $P(A) = \frac{2}{3}$; $P(B) = \frac{5}{9}$; $P(A \cup B) = \frac{4}{5}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{2}{3}+\frac{5}{9}-\frac{4}{5}$

$= \frac{30+25-36}{45}=\frac{19}{45}$

Probability to get both contracts $= \frac{19}{45}$

17. A pair of dice is rolled. What is the probability that they sum to 7 Given that neither die shows a '2'.

Sol. Let A be the event that the sum of the two dice is 7, then

A = {(1, 6), (2, 5), (3, 4); (4, 3), (5, 2), (6, 1)}

Let B the event that neither die shows a '2'

B = {(1, 1), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 3), (3, 4), (3, 5), (3, 6) (4, 1), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 3), (6, 4), (6, 5), (6, 6)}

n(B) = 25

$A \cap B$ = {(1, 6), (3, 4), (4, 3), (6, 1)}

$n(A \cap B) = 4$

Required Probability $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{n(A \cap B)}{n(B)} = \frac{4}{25}$$

SECTION – C

III.18. Find the product of all the values of $(1 + i)^{4/5}$.

Sol. $(1 + i)^{4/5} = \left[\sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)\right]^{4/5}$

$$= 2^{2/5}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{4/5}$$

$$= 2^{2/5}\,(\cos\pi + i\sin\pi)^{1/5}$$

$$= 2^{2/5}\,\text{cis}\left(\frac{2k\pi + \pi}{5}\right) \text{ where } k = 0, 1, 2, 3, 4$$

$$= 2^{2/5}\,\text{cis}\,(2k + 1)\frac{\pi}{5} \text{ where } k = 0, 1, 2, 3, 4$$

$$= 2^{2/5}\,\text{cis}\,\frac{\pi}{5}, 2^{2/5}\,\text{cis}\,\frac{3\pi}{5}, 2^{2/5}\,\text{cis}\,\frac{5\pi}{5}, 2^{2/5}\,\text{cis}\,\frac{7\pi}{5}, 2^{2/5}\,\text{cis}\,\frac{9\pi}{5}$$

Product of all the values of $(1 + i)^{4/5}$

$$= 2^{2/5} \text{cis} \frac{\pi}{5} . 2^{2/5} \text{cis} \frac{3\pi}{5} . 2^{2/5} \text{cis} \frac{5\pi}{5} . 2^{2/5} \text{cis} \frac{7\pi}{5} . 2^{2/5} \text{cis} \frac{9\pi}{5}$$

$$= (2^{2/5})^5 \text{ cis}\left(\frac{\pi}{5} + \frac{3\pi}{5} + \frac{5\pi}{5} + \frac{7\pi}{5} + \frac{9\pi}{5}\right)$$

$$= 2^2 \text{ cis } 5\pi = 4(-1) = -4$$

19. Given that the sum of two roots of $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ is zero. Find the roots of the equation.

Sol. Let $\alpha, \beta, \gamma, \delta$ are the roots of given equation, since sum of two is zero.

$\alpha + \beta = 0$

Now $\alpha + \beta + \gamma + \delta = 2 \Rightarrow \gamma + \delta = 2$

Let $\alpha\beta = p, \ \gamma\delta = q$

The equation having the roots α, β is $x^2 - (\alpha + \beta) x + \alpha\beta = 0$

$\therefore x^2 + p = 0$

The equation having the roots γ, δ is $x^2 - (\gamma + \delta)x + \gamma\delta = 0$

$\therefore x^2 - 2x + q = 0$

$\therefore x^4 - 2x^3 + 4x^2 + 6x - 21$

$= (x^2 + p)(x^2 - 2x + q)$

$= x^4 - 2x^3 + x^2 (p + q) - 2px + pq$

Comparing the like terms,

$p + q = 4,$	$-2p = 6$
$-3 + q = 4$	$p = -3$ and $q = 7$

$\therefore x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$ and $x^2 - 2x + 7 = 0$

$$\Rightarrow x = \frac{2 \pm \sqrt{4 - 28}}{2} = \frac{2 \pm 2\sqrt{6}i}{2}$$

$$= 1 \pm \sqrt{6}i$$

$\therefore$ Roots are $-\sqrt{3}, \sqrt{3}, 1 - i\sqrt{6}$ and $1 + i\sqrt{6}$

20. If I, n are positive integers, 0 < f < 1 and if $(7+4\sqrt{3})^n$ = I + f, then show that i) I is an odd integer and ii) (I + f) (I – f) = 1.

Sol. Given I, n are positive integers and

$(7+4\sqrt{3})^n = I + f, 0 < f < 1$

Let $7-4\sqrt{3} = F$.

Now $6 < 4\sqrt{3} < 7 \Rightarrow -6 > -4\sqrt{3} > -7$

$\Rightarrow 1 > 7-4\sqrt{3} > 0 \Rightarrow 0 < (7-4\sqrt{3})^n < 1$

$\therefore$ 0 < F < 1

$I + f + F = (7+4\sqrt{3})^n + (7-4\sqrt{3})^n = \left[{}^nC_0\, 7^n + {}^nC_1\, 7^{n-1}(4\sqrt{3}) + {}^nC_2\, 7^{n-2}(4\sqrt{3})^2 + ... + {}^nC_n\,(-4\sqrt{3})^n\right]$

$= \left[{}^nC_0\, 7^n - {}^nC_1\, 7^{n-1}(4\sqrt{3}) + {}^nC_2\, 7^{n-2}(4\sqrt{3})^2 + ... + {}^nC_n\,(-4\sqrt{3})^n\right]$

$= 2\left[{}^nC_0\, 7^n + {}^nC_2\, 7^{n-2}(4\sqrt{3})^2 + ...\right]$

= 2k where k is an integer.

$\therefore$ I + f + F is an even integer.

$\Rightarrow$ f + F is an integer since I is an integer.

But 0 < f < 1 and 0 < F < 1 $\Rightarrow$ 0 < f + F < 2

$\therefore$ f + F = 1 .... (1)

$\Rightarrow$ I + 1 is an even integer.

$\therefore$ I is an add integer.

$\Rightarrow$ (I + f) (I – f) = (I + f) F, By (1)

$= (7+4\sqrt{3})^n (7-4\sqrt{3})^n$

$= \left[(7+4\sqrt{3})(7-4\sqrt{3})\right]^n$

$= (49-48)^n = 1$

21. Find the First 3 terms in the expansion of i) $(3 + 5x)^{-7/3}$ ii) $(1 + 4x)^{-4}$

Sol. **(i)** $(3 + 5x)^{-7/3} = \left[3\left(1+\frac{5}{3}x\right)\right]^{-7/3} = (3)^{-7/3}\left(1+\frac{5}{3}x\right)^{-7/3}$

Now we have $(1 + x)^{-p/q} = 1 - \frac{p}{1!}\left(\frac{x}{q}\right) + \frac{p(p+q)}{1.2}\left(\frac{x}{q}\right)^2 + ...$

Here p = 7, q = 3, $\frac{x}{q} = \frac{\frac{5}{3}x}{3} = \frac{5}{9}x$

$\therefore (3 + 5x)^{-7/3} = (3)^{-7/3}\left[1 - \frac{7}{1!}\left(\frac{5}{9}x\right) + \frac{(7)(10)}{1.2}\left(\frac{5}{9}x\right)^2 + ...\right]$

$= (3)^{-7/3}\left[1 - \frac{35}{9}x + \frac{875}{81}x^2 - ....\right]$

$\therefore$ The first 3 terms of $(3 + 5x)^{-7/3}$ are

$3^{-7/3}, \frac{-3^{7/3}.35x}{9}, 3^{-7/3}\frac{875}{81}.x^2$

ii) We have $(1 + x)^{-n} = 1 - nx + \frac{(n)(n+1)}{1.2}x^2 + ...$

Here n = 4, x = 4x

$\therefore (1 + 4x)^{-4} = 1 - 4(4x) + \frac{(4)(5)}{1.2}.(4x)^2 + ...$

$= 1 - 16x + 160x^2 - ....$

$\therefore$ The first 3 terms of $(1 + 4x)^{-4}$ are 1, –16x, $160x^2$.

22. Find the mean deviation about the median for the following distribution.

Class interval	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**
	50 – 60	**60 – 70**	**70 – 80**		
Frequency	**5**	**8**	**7**	**12**	**28**
	20	**10**	**10**		

Sol. Construct the table $\frac{N}{2} = \frac{100}{2} = 50$

Observation lies in the interval 40 – 50

This is the median class

$$\therefore \text{ Median } = L + \left\{\frac{\frac{N}{2} - P.c.f}{f}\right\} \times i$$

$$= 40 + \left\{\frac{50-32}{28}\right\} \times 10 = 40 + 6.43 = 46.43$$

Class interval	**frequency f_i**	**Cumulative frequency**	**Mid point x_i**	**$\lvert x_i$ – median$\rvert$**	**$f_i \lvert x_i$ – Median$\rvert$**
0 – 10	5	5	5	41.43	207.15
10 – 20	8	13	15	31.43	251.44
20 – 30	7	20	25	21.43	150.01
30 – 40	12	32	35	11.43	137.16
40 – 50	28	60	45	1.43	40.04
50 – 60	20	80	55	8.57	171.40
60 – 70	10	90	65	18.57	185.70
70 – 80	10	100	75	28.57	285.70
	$N = \Sigma f_i = 100$				1428.60

$$\therefore \text{ Mean deviation about Median } = \frac{1}{N}\sum_{i=1}^{8} f_i \,|\, x_i - \text{median}|$$

$$= \frac{1}{100}(1428.6) = 14.286$$

23. A, B, C are aiming to shoot a balloon, A will succeed 4 times out of 5 attempts. The chance of B to shoot the balloon is 3. Out of 4 and that of C is 2 out of 3. If the three aim the balloon simultaneously, then find the probability that atleast two of them hit the balloon.

Sol. Given $P(A) = \frac{4}{5}$, $P(B) = \frac{3}{4}$, $P(C) = \frac{2}{3}$

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{4}{5} = \frac{1}{5}$$

$$P(\bar{B}) = 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P(\bar{C}) = 1 - P(C) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$P(A \cap B \cap \bar{C}) = P(A)\,P(B)\,P(\bar{C}) = \frac{4}{5} . \frac{3}{4} . \frac{1}{3} = \frac{1}{5}$$

$$P(A \cap \bar{B} \cap C) = P(A)\,P(\bar{B})\,P(C) = \frac{4}{3} . \frac{1}{4} . \frac{2}{3} = \frac{2}{15}$$

$$P(\bar{A} \cap B \cap C) = P(\bar{A})\,P(B)\,P(C) = \frac{1}{5} . \frac{3}{4} . \frac{2}{3} = \frac{1}{10}$$

$$P(A \cap B \cap C) = P(A)\,P(B)\,P(C) = \frac{4}{5} . \frac{3}{4} . \frac{2}{3} = \frac{2}{5}$$

Probability that atleast two of them hit the balloon =

$$\frac{1}{5} + \frac{2}{15} + \frac{1}{10} + \frac{2}{3} = \frac{25}{30} = \frac{5}{6}$$

24. The probability of a bomb hitting a bridge $\frac{1}{2}$ and three direct hits (not necessarily consecutive) are needed to destroy it. Find the minimum number of bombs required so that the probability of the bridge being destroyed is greater than 0.9.

Sol. Let n be the minimum number of bombs required and X be the number of bombs that hit the bridge, then X follows binomial distribution with parameters n and p = $\frac{1}{2}$

Now $P(X \geq 3) > 0.9$

$\Rightarrow 1 - P(X < 3) > 0.9$

$\Rightarrow P(X < 3) < 0.1$

$\Rightarrow P(X = 0) + P(X = 1) + P(X = 2) < 0.1$

$$\Rightarrow {}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n + {}^nC_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1} + {}^nC_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2} < 0.1$$

$$\Rightarrow 1.\frac{1}{2^n} + \frac{n}{2^n} + \frac{n(n-1)}{2}\frac{1}{2^n} < \frac{1}{10}$$

$$\Rightarrow \frac{1}{2^n} + \frac{n}{2^n} + \frac{n^2-n}{2.\,2^n} < \frac{1}{10}$$

$$\Rightarrow \frac{1}{2^n}\left(1+n+\frac{n^2-n}{2}\right) < \frac{1}{10}$$

$$\Rightarrow \frac{1}{2^n}\left(\frac{2+2n+n^2-n}{2}\right) < \frac{1}{10}$$

$\Rightarrow 5(n^2 + n + 2) < 2^n$

By trial and error, we get $n \geq 9$

$\therefore$ The least value of n is 9

$\therefore n = 9$

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