PRACTICE PAPER – 5

SOLUTIONS

SECTION – A

I. 1. Write the multiplicative inverse of (sin θ, cos θ)

Sol. Multiplicative inverse of (sin θ, cos θ) is

$$= \left(\frac{\sin\theta}{\sin^2\theta + \cos^2\theta}, \frac{-\cos\theta}{\sin^2\theta + \cos^2\theta} \right)$$

$= (\sin\theta, -\cos\theta)$

2. If 4x + i (3x – y) = 3 – 6i where x and y are real numbers, then find the values of x and y.

Sol. $\because 4x + i(3x - y) = 3 - 6i$

Equating real and imaginary parts, we get

$4x = 3$ and $3x - y = -6$

$\Rightarrow x = 3/4$ and $3\left(\frac{3}{4}\right) - y = -6$

$\frac{9}{4} + 6 = y$

$\Rightarrow y = \frac{33}{4}$

$\therefore x = \frac{3}{4}$ and $y = \frac{33}{4}$

3. If the cube roots of unity are 1, ω, ω², then find the roots of the equation $(x - 1)^3 + 8 = 0$.

Sol. Given $(x - 1)^3 + 8 = 0$

$\Rightarrow (x - 1)^3 = -8 \Rightarrow (x - 1)^3 = (-2)^3 (1)^3$

$\Rightarrow (x - 1) = (-2)(1)^{1/3}$

$\Rightarrow x - 1 = -2, -2\omega, -2\omega^2$

$\Rightarrow x = 1 - 2, 1 - 2\omega, 1 - 2\omega^2$

$\Rightarrow x = -1, 1 - 2\omega, 1 - 2\omega^2$

4. If $x^2 - 6x + 5 = 0$ and $x^2 - 3ax + 35 = 0$ have a common root, then find *a*.

Sol. The roots of the equation

$x^2 - 6x + 5 = 0$ are

$(x - 1)(x - 5) = 0$

$\Rightarrow x = 1, x = 5$

Case (i) : $x = 1$ is a common root then it is also root for the equation $x^2 - 3ax + 35 = 0$

$\Rightarrow 1 - 3a(1) + 35 = 0$

$\Rightarrow a = \frac{36}{3} = 12$

Case (ii) : $x = 5$ is a common root then

$(5)^2 - 3a(5) + 35 = 0$

$\Rightarrow 60 - 15a = 0 \Rightarrow a = 4$

$\therefore a = 12$ (or) $a = 4$

5. Form the polynomial equation whose roots are the squares of the roots of $x^3 + 3x^2 - 7x + 6 = 0$

Sol. Given equation is $f(x) = x^3 + 3x^2 - 7x + 6 = 0$

Required equation is $f(\sqrt{x}) = 0$

$\Rightarrow x\sqrt{x} + 3x - 7\sqrt{x} + 6 = 0 \Rightarrow \sqrt{x}(x - 7) = -(3x + 6)$

Squaring on both sides,

$\Rightarrow x(x - 7)^2 = (3x + 6)^2$

$\Rightarrow x(x^2 - 14x + 49) = 9x^2 + 36 + 36x$

$\Rightarrow x^3 - 14x^2 + 49x - 9x^2 - 36x - 36 = 0$

i.e. $x^3 - 23x^2 + 13x - 36 = 0$

6. Find the number of 5 letter words that can be formed using the letters of the word 'MIXTURE' which begin with an vowel when repetitions are allowed.

Sol. The word MIXTURE has 7 letters 3 vowels {E, I, U} and 4 consonants {E, M, R, X} we have to fill up 5 blanks.

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Fill the first place with one of the 3 vowels in 3 ways.

Each of the remain 4 places can be filled in 7 ways (since repetition is allowed)

$\therefore$ The number of 5 letter words

$= 3 \times 7 \times 7 \times 7 \times 7 = 3 \times 7^4$

7 If $^{17}C_{2t+1} = {}^{17}C_{3t-5}$, find t.

Sol. $^{17}C_{2t+1} = {}^{17}C_{3t-5}$

$\Rightarrow 2t + 1 = 3t - 5$ or $(2t + 1) + (3t - 5) = 17$

$\Rightarrow 1 + 5 - t$ or $5t = 21$

$\Rightarrow t = 6$ or $t = \dfrac{21}{5}$ which is not an integer

$\therefore t = 6.$

8. Find an approximate value of the following corrected to 4 decimal places. $\sqrt[5]{32.16}$

Sol. $\sqrt[5]{32.16} = (32 + 0.16)^{1/5}$

$= (32)^{1/5} \cdot \left(1 + \dfrac{0.16}{32}\right)^{1/5}$

$= 2\,[1 + 0.005]^{1/5}$

$= 2\left[1 + \dfrac{1}{5}(0.005) + \dfrac{\frac{1}{5}\left(-\frac{4}{5}\right)}{2!}(0.005)^2 + ...\right]$

$= 2\left[1 + 0.001 - \dfrac{2}{25}(0.000025) + ...\right]$

$\simeq 2(1.001) \simeq 2.002$

$\therefore \sqrt[5]{32.16} \simeq 2.002$

9. In the experiment of throwing a die, consider the following events : A = {1, 3, 5}, B = {2, 4}, C = {6} Are these events mutually exclusive ?

Sol. Since the happening of one of the given events A, B, C prevents the happening of other two, hence the given events are mutually exclusive.

Otherwise $A \cap B = \phi$, $B \cap C = \phi$, $C \cap A = \phi$

Hence they are mutually exclusive events.

10. For a binomial distribution with mean 6 and variance 2, find the first two terms of the distribution.

Sol. Let n, p be the parameters of a binomial distribution

Mean (np) = 6 ..... (1)

and variance (npq) = 2 ..... (2)

then $\frac{npq}{np} = \frac{2}{6} \Rightarrow q = \frac{1}{3}$

$\because p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$

From (1) n P = 6

$$n\left(\frac{2}{3}\right) = 6$$

$$n = \frac{18}{2} = 9$$

First two terms of the distribution are

$P(X = 0) = {}^9C_0 \left(\frac{1}{3}\right)^9 = \frac{1}{3^9}$ and

$P(X = 1) = {}^9C_1 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right) = \frac{2}{3^7}$

SECTION – B

II.11. The points P, Q denote the complex numbers z_1, z_2 in the Argand diagram. O is the origin. If $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$, show that $\angle POQ = 90°$.

Sol. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

Then $P(x_1\ y_1)$, $Q(x_2, y_2)$, $O(0, 0)$

$\bar{z}_1 = x_1 - iy_1$, $\bar{z}_2 = x_2 - iy_2$

$z_1\bar{z}_2 + z_2\bar{z}_1 = (x_1 + iy_1)(x_2 - iy_2) + (x_2 + iy_2)(x_1 - iy_1) = 0$

$x_1x_2 + y_1y_2 - ix_1y_2 + ix_2y_1 + x_1x_2 + y_1y_2 - ix_2y_1 + ix_1y_2 = 0$

$2(x_1x_2 + y_1y_2) = 0$

$x_1x_2 + y_1y_2 = 0$

$x_1x_2 = -y_1y_2$

$$\left(\frac{-x_1}{y_1}\right)\left(-\frac{x_2}{y_2}\right) = -1$$

Slope of OP x Slope of OQ = –1

$\therefore$ OP, OQ are perpendicular

$\Rightarrow \angle POQ = 90°$

12. Solve the inequation $\sqrt{x+2} > \sqrt{8-x^2}$.

Sol. When a, b $\in$ R and $a \geq 0, b \geq 0$ then $\sqrt{a} > \sqrt{b} \Leftrightarrow a > b \geq 0$

$\therefore \sqrt{x+2} > \sqrt{8-x^2}$

$\Leftrightarrow x + 2 > 8 - x^2 \geq 0$ and $x > -2$, $|x| < 2\sqrt{2}$

We have $(x + 2) > 8 - x^2$

$\Leftrightarrow x^2 + x - 6 > 0$

$\Leftrightarrow (x + 3)(x - 2) > 0$

$\Leftrightarrow x \in (-\infty, -3) \cup (2, \infty)$

We have $8 - x^2 \geq 0$

$\Leftrightarrow x^2 \le 8 \Leftrightarrow |x| < 2\sqrt{2}$

$\Leftrightarrow x \in [-2\sqrt{2}, 2\sqrt{2}]$

Also $x + 2 > 0 \Leftrightarrow x > -2$

Hence $x + 2 > 8 - x^2 \ge 0$

$\Leftrightarrow x \, \varepsilon \, ((-\infty, -3) \cup (2, \infty)) \cap [-2\sqrt{2}, 2\sqrt{2}]$

and $x > -2$

$\Leftrightarrow x \in (2, 2\sqrt{2})$

$\therefore$ The solution set is $[x \in R : -2 < x \le 2\sqrt{2}]$

13. Solve the equations $6x^3 - 11x^2 + 6x - 1 = 0$, given that the roots of each are in H.P.

Sol. Given equation is $6x^3 - 11x^2 + 6x - 1 = 0$ —— (1)

Put $y = \frac{1}{x}$ so that $\frac{6}{y^3} - \frac{11}{y^2} + \frac{6}{y} - 1 = 0$

$6 - 11y + 6y^2 - y^3 = 0$

$y^3 - 6y^2 + 11y - 6 = 0$ —— (2)

Roots of (1) are in H.P. $\Rightarrow$ Roots of (2) are in A.P.

Let $a - d$, a, $a + d$ be the roots of (2)

Sum $= a - d + a + a + d = 6$

$3a = 6 \Rightarrow a = 2$

Product $= a(a^2 - d^2) = 6$

$2(4 - d^2) = 6$

$4 - d^2 = 3$

$\Rightarrow d^2 = 1, \quad d = 1$

$a - d = 2 - 1 = 1, a = 2, a + d = 2 + 1 = 3$

The roots of (2) are 1, 2, 3

The roots of (1) are $1, \frac{1}{2}, \frac{1}{3}$

14. Find the number of ways of arranging the letters of the word MONDAY so that no vowel occupies even place.

Sol. In the word MONDAY there are two vowels, 4 consonants and three even places, three odd places.

Since no vowel occupies even place, the two vowels can be filled in the three odd places in ${}^3P_2$ ways. The 4 consonants can be filled in the remaining 4 places in 4! ways.

$\therefore$ The number of required arrangements

$= {}^3P_2 \times 4! = 6 \times 24 = 144$

15. Find the number of ways of arranging 6 red roses and 3 yellow roses of different sizes into a garland. In how many of them (i) all the yellow roses are together (ii) no two yellow roses are together

Hint : The number of circular permutations like the garlands of flowers, chains of beads etc., of n things $= \frac{1}{2}(n-1)!$

Sol. Total number of roses = 6 + 3 = 9

$\therefore$ The number of ways of arranging 6 red roses and 3 yellow roses of different sizes into a garland $= \frac{1}{2}(9-1)! = \frac{1}{2} \times 8!$

$= \frac{1}{2} \times 40{,}320 = 20{,}160$

i) Treat all the 3 yellow roses as one unit. Then we have 6 red roses and one unit of yellow roses. They can be arranged in garland form in (7 – 1)! = 6! ways. Now, the 3 yellow roses can be arranged among themselves in 3! ways.

But in the case of garlands, clockwise arrangements look alike.

$\therefore$ The number of required arrangements $= \frac{1}{2} \times 6! \times 3!$

$= \frac{1}{2} \times 720 \times 6 = 2160$

ii) First arrange the 6 red roses in garland form in 5! ways. Then we can find 6 gaps between them. The 3 yellow roses can be arranged in these 6 gaps in ${}^6P_3$ ways.

But in the case of garlands, clock–wise and anti–clockwise arrangements look alike.

$\therefore$ The number of required arrangements $= \frac{1}{2} \times 5! \times {}^6P_3$

$= \frac{1}{2} \times 120 \times 6 \times 5 \times 4 = 7200$

16. Find the coefficient of x^n in the power series expansion of $\frac{x}{(x-1)^2(x-2)}$ specifying the region in which the expansion is valid.

Sol. $\frac{x}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$

Multiplying with $(x-1)^2(x-2)$

$x = A(x-1)(x-2) + B(x-2) + C(x-1)^2$

Put $x = 1$, $1 = B(-1) \Rightarrow B = -1$

Put $x = 2$, $2 = C(1) \Rightarrow C = 2$

Put $x = 0$, $0 = 2A - 2B + C \Rightarrow 2A = 2B - C$

$= -2 - 2 = -4 \Rightarrow A = -2$

$\therefore \frac{x}{(x-1)^2(x-2)} = \frac{-2}{x-1} - \frac{1}{(x-1)^2} + \frac{2}{x-2}$

$= \frac{2}{1-x} - \frac{1}{(1-x)^2} + \frac{2}{-2\left(1-\frac{x}{2}\right)}$

$= 2(1-x)^{-1} - (1-x)^{-2} - \left(1-\frac{x}{2}\right)^{-1}$

$= 2[1 + x + x^2 + \ldots + x^n + \ldots] - [1 + 2x + 3x^2 + \ldots + (n+1)x^n + \ldots]$

$$-\left[1+\frac{x^2}{2}+\frac{x^2}{4}+\ldots.+\frac{x^n}{2^n}+\ldots.\right]$$

$\therefore$ Coefficient of $x^n = 2(1) - (n+1) - \left(\frac{1}{2^n}\right)$

$= 2 - n - 1 - \frac{1}{2^n} = 1 - n - \frac{1}{2^n}$.

17. Find the probability that a non – leap year contains i) 53 Sundays ii) 52 Sundays only.

Sol. A non - leap year contains 365 days 52 weeks and 1 day more.

i) We get 53 sundays when the remaining day is Sunday.

Number of days in the week = 7

$\therefore$ n(S) = 7

Number of ways getting 53 Sundays. n(E) = 1

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{7}$

$\therefore$ Probability of getting 53 Sundays = $\frac{1}{7}$

ii) Probability of getting 52 Sundays $P(\bar{E}) = 1 - P(E)$

$= 1 - \frac{1}{7} = \frac{6}{7}$.

SECTION – C

III.18. Find all the roots of the equation $x^{11} - x^7 + x^4 - 1 = 0$.

Sol. $x^{11} - x^7 + x^4 - 1 = 0$

$\Rightarrow x^7 (x^4 - 1) + 1 (x^4 - 1) = 0$

$\Rightarrow (x^4 - 1)(x^7 + 1) = 0$

Case (i) : $x^4 - 1 = 0$

$x^4 = 1 = (\cos 0 + i \sin 0)$

$\Rightarrow x^4 = (\cos 2k\pi + i \sin 2k\pi)$

$\therefore x = (\cos 2k\pi + i \sin 2k\pi)^{1/4}$

$\Rightarrow x = \text{cis}\left(\frac{2k\pi}{4}\right) = \text{cis}\,\frac{k\pi}{2}$, k = 0, 1, 2, 3.

Case (ii) : $x^7 + 1 = 0$

$\Rightarrow x^7 = -1 = \cos\pi + i\sin\pi$

$\Rightarrow x^7 = \cos(2k\pi + \pi) + i\sin(2k\pi + \pi)$

$\therefore x = [\cos(2k + 1)\pi + i\sin(2k + 1)\pi]^{1/7}$

$\Rightarrow x = \text{cis}(2k + 1)\frac{\pi}{7}$, k = 0 ,1, 2, 3, 4, 5, 6

The values of x are

$\text{cis}\,0 = 1, \text{cis}\frac{\pi}{2} = i, \text{cis}\,\pi = -1, \text{cis}\frac{3\pi}{2} = -i$

$\text{cis}\frac{\pi}{7}, \text{cis}\frac{3\pi}{7}, \text{cis}\frac{5\pi}{7}, \text{cis}\,\pi = -1$

$= -1, \text{cis}\frac{9\pi}{7}, \text{cis}\frac{11\pi}{7}, \text{cis}\frac{13\pi}{7}$

i.e, $\pm i, \pm 1, \text{cis}\frac{\pi}{7}, \text{cis}\frac{3\pi}{7}, \text{cis}\frac{5\pi}{7}, \text{cis}\frac{9\pi}{7}, \text{cis}\frac{11\pi}{7}$ and $\text{cis}\frac{13\pi}{7}$

19. Solve the following equations $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

Sol. This is standard reciprocal equation. Dividing with x^2

$x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$

$\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 26 = 0$ ....(1)

put $a = x + \frac{1}{x}$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = a^2 - 2$$

Substituting in (1)

$a^2 - 2 - 10\,a + 26 = 0$

$\Rightarrow a^2 - 10a + 24 = 0 \Rightarrow (a - 4)(a - 6) = 0$

$a = 4$ or 6

Case (i) $a = 4$

$$x + \frac{1}{x} = 4$$

$\Rightarrow x^2 + 1 = 4x \Rightarrow x^2 - 4x + 1 = 0$

$$\Rightarrow x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

Case (ii) $a = 6$

$$x + \frac{1}{x} = 6$$

$x^2 + 1 = 6x$

$x^2 - 6x + 1 = 0$

$$x = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm 4\sqrt{2}}{2}$$

$$x = \frac{2(3 \pm 2\sqrt{2})}{2} = 3 \pm 2\sqrt{2}$$

$\therefore$ The roots are $3 \pm 2\sqrt{2}, 2 \pm \sqrt{3}$

20. If n is a positive integer and x is any nonzero real number, then prove that $C_0 + C_1\frac{x}{2} + C_2.\frac{x^2}{3} + C_3.\frac{x^3}{4} + \ldots\ldots + C_n.\frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$

Sol. $C_0 + \frac{C_1}{2}x + \frac{C_2}{3}x^2 + \ldots\ldots + \frac{C_n}{n+1}x^n$

$$= {}^nC_0 + \frac{1}{2}{}^nC_1\, x + \frac{1}{3}{}^nC_2\, x^2 + ... + \frac{1}{n+1}.\, {}^nC_n\, x^n$$

$$= 1 + \frac{n}{1!}\frac{x}{2} + \frac{n(n-1)}{2!}.\frac{x^2}{3} + ...$$

$$= 1 + \frac{n}{2!}x^1 + \frac{n(n-1)}{3!}.\, x^2 + ...$$

$$= \frac{1}{(n+1)x}\left[\frac{(n+1)\, x^1}{1!} + \frac{(n+1)n}{2!}x^2 + \frac{(n+1)n\,(n-1)}{3!}x^3 + ...\right]$$

$$= \frac{1}{(n+1)x}\left[{}^{(n+1)}C_1\, x + {}^{(n+1)}C_2\, x^2 + {}^{(n+1)}C_3\, x^3 + ...\right]$$

$$= \frac{1}{(n+1)x}\left[1 + {}^{n+1}C_1\, x + {}^{n+1}C_2\, x^2 + ... + {}^{n+1}C_{n+1}\, x^{n+1} - 1\right]$$

$$= \frac{1}{(n+1)x}\,[(1+x)^{n+1} - 1]$$

$$\therefore C_0 + C_1.\frac{x}{2} + C_2.\frac{x^2}{3} ..... + C_n.\frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)\, x}$$

21. By neglecting x^4 and higher powers of x, find an approximate value of $\sqrt[3]{x^2+64} - \sqrt[3]{x^2+27}$.

Sol. $\sqrt[3]{x^2+64} - \sqrt[3]{x^2+27} = (64 + x^2)^{1/3} - (27 + x^2)^{1/3}$

$$= (64)^{1/3}\left(1 + \frac{x^2}{64}\right)^{1/3} - (27)^{1/3}\left(1 + \frac{x^2}{27}\right)^{1/3}$$

$$= 4\left(1 + \frac{x^2}{192}\right) - 3\left(1 + \frac{x^2}{81}\right),$$

(By neglecting x^4 and higher powers of x)

$$= 4 + \frac{x^2}{48} - 3 - \frac{x^2}{27} = 1 + \frac{(27-48)}{48 \times 27}x^2$$

$$= 1+\left(\frac{-21}{48\times 27}\right)x^2 = 1-\frac{7x^2}{432} \Rightarrow 1-\frac{7}{432}x^2$$

$$\therefore \sqrt[3]{x^2+64} - \sqrt[3]{x^2+27} = 1-\frac{7}{432}x^2$$

22. If each of the observations $x_1, x_2 ..... x_n$ is increased by k, where k is a positive or negative number, then show that the variance remains unchanged.

Sol. Let $\bar{x}$ be the mean of $x_1, x_2 ..... x_n$. Then their variance is given by $\sigma_1^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

If to each observation we add a constant k, then the new (changed) observations will be $y_i = x_i + k$

Then the mean of the new observations $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i + k) = \frac{1}{n}\left[\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} k\right] = \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{1}{n}(nk) = \bar{x} + k$$

The variance of the new observations $= \sigma_2^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i + k - \bar{x} - k)^2 \text{ , using (1) and (2) } = \frac{1}{n}(x_i - \bar{x})^2 = \sigma_2^1$$

23. A fair die is rolled. Consider the events. A = {1, 3, 5},B ={2, 3} and C = {2, 3, 4, 5}. Find i) $P(A \cap B)$, $P(A \cup B)$

ii) $P\left(\frac{A}{B}\right), P\left(\frac{B}{A}\right)$ iii) $P\left(\frac{A}{C}\right), P\left(\frac{C}{A}\right)$ iv) $P\left(\frac{B}{C}\right), P\left(\frac{C}{B}\right)$

Sol. A fair die is rolled $P(A) = \frac{3}{6} = \frac{1}{2}$

$$P(B) = \frac{2}{6} = \frac{1}{3}$$

$$P(C) = \frac{4}{6} = \frac{2}{3}$$

$$n(S) = 6^1 = 6$$

Givne $A = \{1, 3, 5\}, B = \{2, 3\}, C = \{2, 3, 4, 5\}$

i) $A \cap B = \{3\}$ $P(A \cap B) = P\{3\} = \frac{1}{6}$

$\therefore P(A \cap B) = \frac{1}{6}$

$(A \cup B) = \{1, 2, 3, 5\}$

$n(A \cup B) = 4$

$n(S) = 6$

$P(A \cup B) = \frac{4}{6} = \frac{2}{3}$

ii) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$

iii) $P\left(\frac{A}{C}\right) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{2}{6}}{\frac{4}{6}} = \frac{2}{4} = \frac{1}{2}$

$\because A \cap C = \{3, 5\}$

$P\left(\frac{C}{A}\right) = \frac{P(A \cap C)}{P(A)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$

iv) $P\left(\frac{B}{C}\right) = \frac{P(B \cap C)}{P(C)} = \frac{\frac{2}{6}}{\frac{4}{6}} = \frac{1}{2}$

$\because B \cap C = \{2, 3\}$

$P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)} = \frac{\frac{2}{6}}{\frac{2}{6}} = 1$

24. A random variable X has the following probability distribution.

X = x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
P(X = x)	**0**	**k**	**2k**	**2k**	**3k**	**k^2**	**$2k^2$**	**$7k^2 + k$**

Find i) k ii) the mean and iii) P(0 < X < 5).

Sol. Sum of the probabilities = 1

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 10k^2 + 9k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow 10k^2 + 10k - k - 1 = 0$

$\Rightarrow 10k(k + 1) - 1(k + 1) = 0$

$\Rightarrow (10k - 1)(k + 1) = 0$

$k = \frac{1}{10}, -1$

Since $k > 0$

$\therefore k = \frac{1}{10}$

i) $k = \frac{1}{10}$

ii) Mean = $\sum_{i=1}^{n} x_i P(x = x_i)$

Mean (μ) = 0(0) + 1(k) + 2(2k) + 3(2k) +

+ 4(3k) + 5(k^2) + 6($2k^2$) + 7 ($7k^2$ + k)

= 0 + k + 4k + 6k + 12k + $5k^2$ + $12k^2$ + $49k^2$ + 7k

= $66k^2$ + 30k

$$= 66\left(\frac{1}{100}\right) + 30 \times \left(\frac{1}{10}\right)$$

= 0.66 + 3 = 3.66

iii) P(0 < x < 5)

P(0 < x < 5) = P (X = 1) + P(X = 2) + P(X = 3) + P(X = 4)

= k + 2k + 2k + 3k

= 8k

$$= 8\frac{1}{10}$$

$$= \frac{8}{10} = \frac{4}{5}$$

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