PRACTICE PAPER – 6

SOLUTIONS

SECTION – A

I. 1. If $z = 2 - i\sqrt{7}$, then show that $3z^3 - 4z^2 + z + 88 = 0$.

Sol. Given $z = 2 - i\sqrt{7}$

$\Rightarrow z - 2 = -i\sqrt{7}$

$\Rightarrow (z-2)^2 = (-i\sqrt{7})^2$

$\Rightarrow z^2 - 4z + 4 = 7i^2$

$\Rightarrow z^2 - 4z + 4 = -7$

$\Rightarrow z^2 - 4z + 11 = 0$

$3z^3 - 4z^2 + z + 88 = 3z(z^2 - 4z + 11) + 8z^2 - 32z + 88$

$= 3z(0) + 8(z^2 - 4z + 11) = 0 + 8(0) = 0$

$\therefore 3z^3 - 4z^2 + z + 88 = 0$

2. Find the equation of the straight line joining the point – 9 + 6i, 11 – 4i in the Argand plane.

Sol. Given points are – 9 + 6i, 11 – 4i

Let A = (– 9, 6); B = (11, – 4)

Equation of the straight line

$\overline{AB}$ is $y - 6 = \dfrac{-4-6}{11+9}(x+9)$

$y - 6 = \dfrac{-1}{2}(x+9)$

$2y - 12 = -x - 9 \Rightarrow x + 2y - 3 = 0$

3. Find the number of 15^{th} roots of unity, which are also 25^{th} roots of unity.

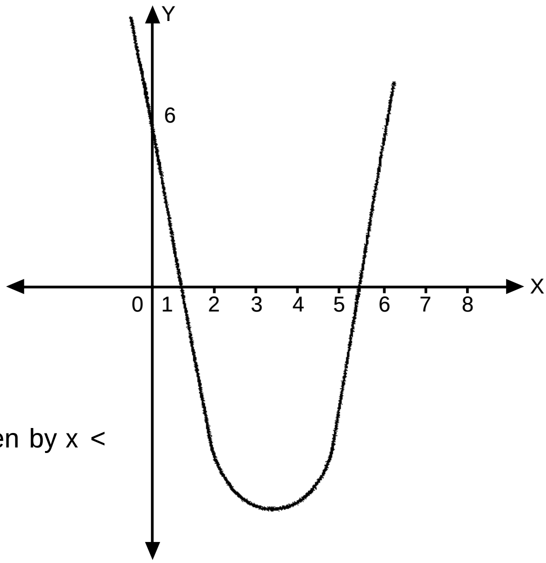
Sol. The number of common roots = H.C.F of {15, 25}

= 5

4. Solve the following inequations by graphical method.
$x^2 - 7x + 6 > 0$

Sol. $f(x) = x^2 - 7x + 6$

x	0	1	2	3	4	5	6	7	8
f(x)	6	0	–4	–6	–6	–4	0	6	14



$f(x) > 0 \Rightarrow y > 0$

Solutions are given by $x < 1$ and $x > 6$

5. Form the polynomial equation whose roots are cubes of the roots of $x^3 + 3x^2 + 2 = 0$

Sol. Given equation is $x^3 + 3x^2 + 2 = 0$

Put $y = x^3$ so that $x = y^{1/3}$

$\therefore\ y + 3y^{2/3} + 2 = 0$

$3y^{2/3} = -(y + 2)$

Cubing on both sides, $27y^2 = -(y + 2)^3$

$= -(y^3 + 6y^2 + 12y + 8)$

$\therefore\ y^3 + 6y^2 + 12y + 8 = 0$

$\Rightarrow y^3 + 33y^2 + 12y + 8 = 0$

Required equation is $x^3 + 33x^2 + 12x + 8 = 0$

6. Find the number of ways of selecting4 English, 3 Telugu and 2 Hindi books out of 7 English, 6 Telugu and 5 Hindi books.

Sol. The number of ways of selecting

4 English books out of 7 books = ${}^7C_4$

3 Telugu books out of 6 books = ${}^6C_3$

2 Hindi books out of 5 books = ${}^5C_2$

Hence, the number of required ways ${}^7C_4 \times {}^6C_3 \times {}^5C_2 = 35 \times 20 \times 10 = 7000$.

7. In how many ways can the letters of the word CHEESE be arranged so that no two E's come together ?

Sol. The given word contains 6 letters in which one C, one H, 3 E's and one S.

Since no two E's come together, first arrange the remaining 3 letters in 3! ways. Then we can find 4 gaps between them. The 3 E's can be arranged in these 4 gaps in $\frac{{}^4P_3}{3!} = 4$ ways.

$\therefore$ The number of required arrangements = $3! \times 4 = 24$

8. Find the remainder when 2^{2013} is divided by 17.

Sol. We know $2^4 = 16$

The reminder when 2^4 is divided by 17 is – 1 $2^{2013} = (2^4)^{503}.\ 2^1$

$\therefore$ The remainder when 2^{2013} is

divided by 17 is $(-1)^{503}.\ 2$

$= (-1)2 \quad = -2.$

9. Find the mean deviation about the mean for the following data 3, 6, 10, 4, 9, 10

Sol. The arithmetic mean of the given data

$$\bar{x} = \frac{3+6+10+4+9+10}{6} = \frac{42}{6} = 7$$

The absolute values of the deviations are

$|x_i - \bar{x}|$, |3 – 7|, |16 – 7|, |10 – 7|, |4 – 7|, |9 – 7|, |10 – 7| = 4, 1, 3, 3, 2, 3

∴ The mean deviation from the mean

$$= \frac{\sum_{i=1}^{6} |x_i - \bar{x}|}{6} = \frac{4+1+3+3+2+3}{6}$$

$$= \frac{16}{6} = 2.666$$

10. An integer in picked from 1 to 20, both inclusive. Find the probability that it is a prime.

Sol. Let E be the event that the number picked from 1 to 20 is a prime and S be the sample space.

$\therefore n(S) = {}^{20}C_1 = 20$

E = {2, 3, 5, 7, 11, 13, 17, 19}

$n(E) = 8$

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

SECTION – B

II.11. The complex number z has argument θ, $0 < \theta < \frac{\pi}{2}$ and satisfy the equation |z – 3i| = 3. Then prove that $\left(\cot\theta - \frac{6}{z}\right) = i$.

Sol. Let z = cos θ + i sin θ

Given |z – 3i| = 3

⇒ |cos θ + i sin θ) – 3i| = 3

⇒ |cos θ + i(sin θ – 3)| = 3

$\Rightarrow \sqrt{\cos^2\theta + (\sin\theta - 3)^2} = 3$

$\Rightarrow \cos^2\theta + \sin^2\theta - 6\sin\theta + 9 = 9$

$\Rightarrow 1 - 6\sin\theta = 0$

$\Rightarrow 6 \sin\theta = 1$

$\Rightarrow \sin\theta = \frac{1}{6}$

since $0 < \theta < \frac{\pi}{2}$

$\therefore \cos\theta = \frac{\sqrt{35}}{6}$

$\cot\theta = \sqrt{35}$

$\therefore \cot\theta - \frac{6}{z} = \cot\theta - \frac{6}{\cos\theta + i\sin\theta}$

$= \cot\theta - 6(\cos\theta - i\sin\theta)$

$= \sqrt{35} - 6\left[\frac{\sqrt{35}}{6} - i.\frac{1}{6}\right]$

$= \sqrt{35} - \sqrt{35} + i = i$

Hence $\cot\theta - \frac{6}{z} = i$

12. Determine the range of the following expressions. $\mathbf{\frac{x^2 + x + 1}{x^2 - x + 1}}$

Sol. Let $y = \frac{x^2 + x + 1}{x^2 - x + 1}$

$\Rightarrow x^2y - xy + y = x^2 + x + 1$

$\Rightarrow x^2y - xy + y - x^2 - x - 1 = 0$

$\Rightarrow x^2(y - 1) - x(y + 1) + (y - 1) = 0$

x is real $\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow (y + 1)^2 - 4(y - 1)^2 \geq 0$

$\Rightarrow (y + 1)^2 - (2y - 2)^2 \geq 0$

$\Rightarrow (y + 1 + 2y - 2)(y + 1 - 2y + 2) \geq 0$

$\Rightarrow (3y - 1)(-y + 3) \geq 0$

$\Rightarrow -(3y - 1)(y - 3) \geq 0$

a = coeff. of $y^2 = -3 < 0.$, But

The expression ≥ 0

$\Rightarrow$ y lies between $\frac{1}{3}$ and 3

$\therefore$ The range of $\frac{x^2+x+1}{x^2-x+1}$ is $\left[\frac{1}{3}, 3\right]$

13. Find the polynomial equation whose roots are the translates of the equation $3x^5 - 5x^3 + 7 = 0$ by 4.

Sol. Given $f(x) = 3x^5 - 5x^3 + 7 = 0$

Required equation is $f(x - 4) = 0$

$3(x-4)^5 - 5(x-4)^3 + 7 = 0$

– 4	3	0	–5	0	0	7
	0	–12	48	–172	688	–2752
	3	–12	43	–172	688	–2745
	0	–12	96	–556	2912	A_5
	3	–24	139	–728	3600	
	0	–12	144	–1132	A_4	
	3	–36	283	–1860		
	0	–12	192	A_3		
	3	–48	475			
	0	–12	A_2			
	3	60				
	A_0	A_1				

Required equation is

$$x^5 - 60x^4 + 475x^3 - 1860x^2 + 3600x - 2745 = 0$$

14. Find the number of ways of arranging the letters of the word TRIANGLE so that the relative positions of the vowels and consonants are not disturbed.

Sol. *Hint : Vowels – A, E, I, O U* In a given, word, number of vowels is 3

number of consonants is 5

C C V V C C C V

Since the relative positions of the vowels and consonants are not disturbed,

the 3 vowels can be arranged in their relative positions in 3! ways and the 5 consonants can be arranged in their relative positions in 5! ways.

∴ The number of required arrangements = (3!) (5!) = (6) (120) = 720.

15. Find the sum of all 4 – digited numbers that can be formed using the digits 1, 3, 5, 7, 9.

Sol. We know that the number of 4 digited numbers that can be formed using the digits 1, 3, 5, 7, 9 is $^5P_4 = 120$.

We have to find their sum. We first find the sum of the digits in the units place of all the 120 numbers. Put 1 in the units place.

			1

The remaining 3 places can be filled with the remaining 4 digits in $^4P_3$ ways. Which means that there are $^4P_3$ number of 4 digited numbers with 1 in the units place. Similarly, each of the other digits 3, 5, 7, 9 appears in the units place $^4P_3$ times. Hence, by adding all these digits of the units place, we get the sum of the digits in the units place.

$^4P_3 \times 1 + {}^4P_3 \times 3 + {}^4P_3 \times 5 + {}^4P_3 \times 7 + {}^4P_3 \times 9$

$= {}^4P_3 (1 + 3 + 5 + 7 + 9)$

$= {}^4P_3 (25)$.

Similarly, we get the sum of all digits in 10's place also as $^4P_3 \times 25$. Since it is in 10's place, its value is

$^4P_3 \times 25 \times 10.$

Like this the values of the sum of the digits in 100's place and 1000's place are respectively

$^4P_3 \times 25 \times 100$ and $^4P_3 \times 25 \times 1000.$

On adding all these sums, we get the sum of all the 4 digited numbers formed by using the digits 1, 3, 5, 7, 9. Hence the required sum is

$^4P_3 \times 25 \times 1 + {}^4P_3 \times 25 \times 10 + {}^4P_3 \times 25 \times 100 + {}^4P_3 \times 25 \times 1000$

$= {}^4P_3 \times 25 \times 1111$

$= 24 \times 25 \times 1111 = 6{,}66{,}600.\ {}^3P_2$

16. Resolve $\frac{9}{(x-1)(x+2)^2}$ into the partial fraction.

Sol. Let $\frac{9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

Multiplying with $(x-1)(x+2)^2$

$9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$

$x = 1 \Rightarrow 9 = 9A \Rightarrow A = 1$

$x = -2 \Rightarrow 9 = -3C \Rightarrow C = -3$

Equating the coefficients of x^2

$A + B = 0 \Rightarrow B = -A = -1$

$\therefore \frac{9}{(x-1)(x+2)^2} = \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$

17. Prove that A and B are independent events if and only if $P\left(\frac{A}{B}\right) = P\left(\frac{A}{B^c}\right)$

Sol. Let A and B are independent

Then $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(A).P(B)}{P(B)} = P(A)$ .... (1)

$$P\left(\frac{A}{B^c}\right) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{P(A).P(B^c)}{P(B^c)} = P(A) \quad .... (2)$$

From (1) & (2)

$$P\left(\frac{A}{B}\right) = P\left(\frac{A}{B^c}\right)$$

Let $P\left(\frac{A}{B}\right) = P\left(\frac{A}{B^c}\right)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B^c)}{P(B^c)} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

$\Rightarrow P(A \cap B) - P(B)\,P(A \cap B) = P(A)\,P(B) - P(B)\,P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A).\,P(B)$

$\therefore$ A, B are independent.

Hence, A, B are independent iff

$$P\left(\frac{A}{B}\right) = P\left(\frac{A}{B^c}\right)$$

SECTION – C

III.18. If n is an integer then show that

$$\mathbf{(1 + i)^{2n} + (1 - i)^{2n} = 2^{n+1} \cos \frac{n\pi}{2}}.$$

Sol. Let $1 + i = \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$$= 2^{1/2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$(1 + 1)^{2n} = \left(2^{1/2}\right)^{2n}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{2n}$$

$$= 2^n\left(\cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2}\right) \quad —— (1)$$

$$1 - i = \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right) = 2^{1/2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

$$(1-i)^{2n} = \left(2^{1/2}\right)^{2n}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)^{2n}$$

$$= 2^n\left(\cos\left(-\frac{n\pi}{2}\right) + i\sin\left(-\frac{n\pi}{2}\right)\right)$$

$$= 2^n\left(\cos\frac{n\pi}{2} - i\sin\frac{n\pi}{2}\right) \text{ —— (2)}$$

Adding (1), (2)

$$(1+i)^{2n} + (1-i)^{2n} = 2^n\left(\cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2}\right)$$

$$+2^n\left(\cos\frac{n\pi}{2} - i\sin\frac{n\pi}{2}\right)$$

$$= 2^n\left(\cos\frac{n\pi}{2} + i\,.\sin\frac{n\pi}{2} + \cos\frac{n\pi}{2} - i\sin\frac{n\pi}{2}\right)$$

$$= 2^n\left(2\cos\frac{n\pi}{2}\right) = 2^{n+1}.\cos\left(\frac{n\pi}{2}\right)$$

19. Solve $9x^3 - 15x^2 + 7x - 1 = 0$, given that two of its roots are equal.

Sol. Suppose α, β, γ are the roots of $9x^3 - 15x^2 + 7x - 1 = 0$

$$\alpha + \beta + \gamma = \frac{15}{9} = \frac{5}{3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{7}{9}$$

$$\alpha\beta\gamma = \frac{1}{9}$$

Given $\alpha = \beta$ ($\because$ two of its roots are equal)

$$2\alpha + \gamma = \frac{5}{3} \Rightarrow \gamma = \frac{5}{3} - 2\alpha$$

$\alpha^2 + 2\alpha\gamma = \frac{7}{9}$

$\Rightarrow \alpha^2 + 2\alpha\left(\frac{5}{3} - 2\alpha\right) = \frac{7}{9} \Rightarrow \alpha^2 + \frac{2\alpha(5-6\alpha)}{3} = \frac{7}{9}$

$\Rightarrow 9\alpha^2 + 6\alpha(5 - 6\alpha) = 7 \Rightarrow 9\alpha^2 + 30\alpha - 36\alpha^2 = 7$

$\Rightarrow 27\alpha^2 - 30\alpha + 7 = 0 \Rightarrow (3\alpha - 1)(9\alpha - 7) = 0$

$\Rightarrow \alpha = \frac{1}{3}$ or $\frac{7}{9}$

Case i) $\alpha = \frac{1}{3}$

$\gamma = \frac{5}{3} - 2\alpha = \frac{5}{3} - \frac{2}{3} = 1$

The roots are $\frac{1}{3}, \frac{1}{3}, 1$

Case ii) $\alpha = \frac{7}{9}$

$\gamma = \frac{5}{3} - 2\alpha = \frac{5}{3} - \frac{14}{9} = \frac{1}{9}$

$\alpha\beta\gamma = \frac{7}{9} \cdot \frac{7}{9} \cdot \frac{1}{9} \neq \frac{1}{9}$ $\Rightarrow$ does not satisfy the given equation.

The roots are $\frac{1}{3}, \frac{1}{3}, 1$

20. Find the number of irrational terms in the expansion of $(5^{1/6} + 2^{1/8})^{100}$.

Sol. General term

$T_{r+1} = {}^{100}C_r (5^{1/6})^{100-r} (2^{1/8})^r = {}^{100}C_r \, 5^{\frac{100-r}{6}} \cdot 2^{\frac{r}{8}}$

$\frac{100-r}{6}$ is an integer in the span or $0 \le r \le 100$ if r = 4, 10, 16, 22, 28, 34, 40, 46, 52, 58, 64, 70, 76, 82, 88, 94, 100

$\frac{r}{8}$ is an integer in the span of $0 \le r \le 100$ if r = 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96

$\frac{100-r}{6}, \frac{r}{8}$ both an integers

If r = 16, 40, 64, 88

$\therefore$ The number of rational terms in the expansion of $(5^{1/6} + 2^{1/8})^r$ is 4.

$\therefore$ The number of irrational terms in the expansion of $(5^{1/6} + 2^{1/8})^r$ is 101 – 4 = 97 terms.

21. If the coefficients of 4 consecutive terms in the expansion of $(1 + x)^n$ are a_1, a_2, a_3, a_4 respectively, then show that

$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$$

Sol. Given a_1, a_2, a_3, a_4 are the coefficients of 4 consecutive terms in $(1 + x)^n$ respectively.

Let $a_1 = {}^nC_{r-1}$, $a_2 = {}^nC_r$, $a_3 = {}^nC_{r+1}, a_4 = {}^nC_{r+2}$

L.H.S. : $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{a_1}{a_1\left(1+\frac{a_2}{a_1}\right)} + \frac{a_3}{a_3\left(1+\frac{a_4}{a_3}\right)}$

$$= \frac{1}{1+\frac{{}^nC_r}{{}^nC_{r-1}}} + \frac{1}{1+\frac{{}^nC_{r+2}}{{}^nC_{r+1}}}$$

$$= \frac{1}{1+\frac{n-r+1}{r}} + \frac{1}{1+\frac{n-r-1}{r+2}}$$

$$= \frac{r}{n+1} + \frac{r+2}{r+2+n-r-1}$$

$$= \frac{r+r+2}{n+1} = \frac{2(r+1)}{n+1}$$

R.H.S. : $\frac{2a_2}{a_2+a_3} = \frac{2a_2}{a_2\left(1+\frac{a_3}{a_2}\right)} \frac{2}{1+\frac{{}^nC_{r+1}}{{}^nC_r}} = \frac{2}{1+\frac{n-r}{r+1}}$

$= \frac{2(r+1)}{n+1} = \text{L.H.S}$

$\therefore \frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{2a_2}{a_2+a_3}$

22. Find the mean deviation about the mean for the following continuous distribution.

Height (in cms)	**95 - 105**	**105 - 115**	**115 - 125**	**125 - 135**
	135 - 145	**145 - 155**		
Number of boys	**9**	**13**	**26**	**30**
	12	**10**		

Sol. Taking the assumed mean a = 130 and h = 10 construct the table.

Class Interval Hight (in cms)	**No.of Boys frequency f_i**	**mid point**	$d_i = \frac{x_i - 130}{10}$	$f_i d_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
95-105	9	100	–3	–27	25.3	227.7
105-115	13	110	–2	–26	15.3	198.9
115-125	26	120	–1	–26	5.3	137.8
125-135	30	130	0	0	4.7	141.0
135-145	12	140	1	12	14.7	176.4
145-155	10	150	2	20	24.7	247.0
	$N = \Sigma f_i = 100$			–47		1128.8

$\bar{x} = a + \left(\frac{\Sigma f_i d_i}{N}\right)h$

$= 130 + \left(\frac{-47}{100}\right)10$

$= 130 - 4.7 = 125.3$

Mean deviation about the mean $= \frac{1}{N}\sum_{i=1}^{6} f_i \,|\, x_i - \bar{x} \,|$

$= \frac{1}{100}(1428.8) = 14.288$

23. State and prove BAYE'S THEOREM.

Sol. **Statement :** Let $E_1, E_2, \ldots\ldots\ldots., E_n$ are mutually exclusive and exhaustive events or a random experiment with $P(E_i) \neq 0$ for i = 1, 2, 3, ..., n. Then for any event A or the random experiment with $P(A) \neq 0$

$$P\left(\frac{E_k}{A}\right) = \frac{P(E_k)\,P\left(\frac{A}{E_k}\right)}{\sum_{i=1}^{n} P(E_i)\,P\left(\frac{A}{E_i}\right)}, \; k = 1,2, \ldots\ldots,n$$

Proof : Given that $P(E_i) > 0$ for i = 1,2, ..., n By hypothesis $i \neq j$

$E_i \cap K_j = \phi$ and $E_1 \cup E_2 \cup \ldots\ldots.. \cup E_n = S$

$A = A \cap S$

$= A \cap \left(\bigcup_{i=1}^{n} E_i\right)$

$= \bigcup_{i=1}^{n} (A \cap E_i)$

Also for $i \neq j$ $(A \cap E_i) \cap (A \cap E_j) = A \cap (E_i \cap E_j)$

$= A \cap \phi = \phi$

By multiplication theorem

$$P(A) = \sum_{i-1}^{n} P(A \cap E_i) = \sum_{i-1}^{n} P(E_i)\,P\left(\frac{A}{E_i}\right)$$

Hence $P\left(\frac{E_k}{A}\right) = \frac{P(E_k \cap A)}{P(A)} = \frac{P(E_K)\,P\left(\frac{A}{E_k}\right)}{\sum_{i=1}^{n} P(E_i)\,P\left(\frac{A}{E_i}\right)}$

24. The range of a random variable X is {0, 1, 2}. Given that $P(X = 0) = 3c^3$, $P(X = 1) = 4c - 10c^2$, $P(X = 2) = 5c - 1$
i) Find the value of c ii) $P(X < 1)$, $P(1 < X \le 2)$ and $P(0 < X \le 3)$

Sol. $P(X = 0) + P(X = 1) + P(X = 2) = 1$

$3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$3c^3 - 10c^2 + 9c - 2 = 0$

c = 1 satisfy this equation

$c = 1 \Rightarrow P(X = 0) = 3$ which is not possible

Dividing with c – 1, we get $3c^2 - 7c + 2 = 0$

$(c - 2)(3c - 1) = 0$

$c = 2$ or $c = \frac{1}{3}$

$c = 2 \Rightarrow P(X = 0) = 3.2^3 = 24$ which is not possible

$\therefore c = \frac{1}{3}$

i) $P(X < 1) = P(X = 0) = 3.c^3 = \left(\frac{1}{3}\right)^3 = 3.\frac{1}{27} = \frac{1}{9}$

ii) $P(1 < X \le 2) = P(X = 2) = 5c - 1 = \frac{5}{3} - 1 = \frac{2}{3}$

iii) $P(0 < X \le 3) = P(X = 1) + P(X = 2)$

$= 4c - 10c^2 + 5c - 1 = 9c - 10c^2 - 1$

$= 9.\frac{1}{3} - 10.\frac{1}{9} - 1 = 3 - \frac{10}{9} - 1 = 2 - \frac{10}{9} = \frac{8}{9}$

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