# PRACTICE PAPER – 2

## SOLUTIONS

### SECTION – A

**1. Find the chord of contact of (0, 5) with respect to the circle $x^2 + y^2 - 5x + 4y - 2 = 0$**

**Sol.** Equation of the chord of contact is $S_1 = 0$

i.e., $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

Equation of the circle is

$S = x^2 + y^2 - 5x + 4y - 2 = 0$

Equation of the chord of contact is

$$x.\ 0 + y.\ 5 - \frac{5}{2}(x + 0) + 2(y + 5) - 2 = 0$$

Multiplying with 2

$10y - 5x + 4y + 20 - 4 = 0$

$-5x + 14y + 16 = 0$

or $5x - 14y - 16 = 0$

**2. Find the equation of the tangent at the point 30° (parametric value of θ) of the circle is $x^2 + y^2 + 4x + 6y - 39 = 0$.**

**Sol.** Equation of the circle is

$x^2 + y^2 + 4x + 6y - 39 = 0$

$g = 2,\ f = 3,\ r = \sqrt{4 + 9 + 39} = \sqrt{52} = 2\sqrt{3}$

$\theta = 30^\circ$

Equation of the tangent is

$(x + g)\cos 30^\circ + (y + f)\sin 30^\circ = r$

$$(x + 2)\frac{\sqrt{3}}{2} + (y + 3).\frac{1}{2} = 2\sqrt{13}$$

$\sqrt{3}x + 2\sqrt{3} + y + 3 = 4\sqrt{13}$

$\sqrt{3}x + y + \left(3 + 2\sqrt{3} - 4\sqrt{13}\right) = 0$

**3. Find the angle between the circles are $x^2 + y^2 + 6x - 10y - 135 = 0$, $x^2 + y^2 - 4x + 14y - 116 = 0$.**

**Sol.** $C_1 = (-3, 5)$ $\quad$ $C_2 = (2, -7)$

$r_1 = \sqrt{9+25+135}$ $\quad$ $r_2 = \sqrt{4+49+116}$

$r_1 = 13$ $\quad$ $r_2 = 13$

$C_1C_2 = \sqrt{(-3-2)^2 + (5+7)^2}$

$= 13$

$$\cos\theta = \left|\frac{d^2 - r_1^2 - r_2^2}{2r_1r_2}\right|$$

$$= \left|\frac{(13)^2 - (13)^2 - (13)^2}{2\times(13)\,(13)}\right| = \left|\frac{-1}{2}\right|$$

$$\cos\theta = \left|\frac{-1}{2}\right|$$

$$\theta = \frac{2\pi}{3}$$

**4. Show that the line $2x - y + 2 = 0$ is a tangent to the parabola $y^2 = 16x$. Find the point of contact also.**

**Sol.** Given line is $2x - y + 2 = 0$

$\Rightarrow y = 2x + 2$

Comparing with $y = mx + c$ we get $m = 2$,

$c = 2$

Comparing $y^2 = 16x$ with $y^2 = 4ax$

We get $4a = 16 \Rightarrow a = 4$

$$\frac{a}{m} = \frac{4}{2} = 2 = c$$

$$\therefore \text{Point of contact} = \left(\frac{a}{m^2}, \frac{2a}{m}\right) = \left(\frac{4}{2^2}, \frac{2(4)}{2}\right)$$

$= (1, 4)$

**5. Find the equations of the hyperbola whose foci are (±5, 0) the transverse axis is of length 8.(May '11)**

**Sol.** Foci are S(±5,0) $\therefore$ ae = 5

Length of transverse axis = 2a = 8 a = 4

$$e = \frac{5}{4}$$

$$b^2 = a^2 (e^2 - 1) = 16\left(\frac{25}{6} - 1\right) = 9$$

Equation of the hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$$9x^2 - 16y^2 = 144.$$

**6. Evaluate $\int\left(\frac{x^6 - 1}{1 + x^2}\right)$ dx for x ∈ R.**

**Sol.** $\int\left(\frac{x^6 - 1}{1 + x^2}\right)dx = \int\left[(x^4 - x^2 + 1) - \frac{2}{1 + x^2}\right]dx$

$$= \int x^4\, dx - \int x^2\, dx + \int dx - 2\int \frac{dx}{1 + x^2}$$

$$= \frac{x^5}{5} - \frac{x^3}{3} + x - 2\tan^{-1} x + C.$$

**7. $\int \frac{x^2}{\sqrt{1 - x^6}}$ dx on I = (– 1, 1).**

**Sol.** $\int \frac{x^2\, dx}{\sqrt{1 - x^6}}$

$t = x^3 \Rightarrow dt = 3x^2\, dx$

$$\int \frac{x^2\, dx}{\sqrt{1 - x^6}} = \frac{1}{3}\int \frac{dt}{\sqrt{1 - t^2}}$$

$$= \frac{1}{3}\sin^{-1} t + c = \frac{1}{3}\sin^{-1}(x^3) + c$$

**8. Evaluate** $\int_0^a \frac{dx}{x^2 + a^2}$

**Sol.** $\int_0^a \frac{dx}{x^2+a^2} = \left[\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]_0^a$

$= \frac{1}{a}\ [\tan^{-1}(1) - \tan^{-1}(0)]$

$= \frac{1}{a}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{4a}$

**9. Evaluate** $\int_2^3 \frac{2x}{1+x^2}\,dx$

**Sol.** $I = [ln|1 + x^2|]_2^3$

$= ln\ 10 - ln\ 5$

$= ln(10/5)$

$= ln\ 2$

**10. Evaluate** $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

**Sol.** $\frac{dy}{dx} = e^{x-y} + x^2 . e^{-y} = \frac{e^x}{e^y} + \frac{x^2}{e^y}$

$\int e^y . dy = \int (e^x + x^2)\,dx$

Solution is $e^y = e^x + \frac{x^3}{3} + c$

## SECTION – B

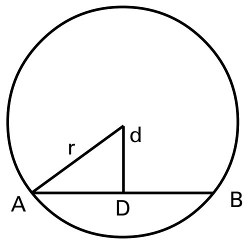
**11. Find the equation of the circle with centre (–2, 3) cutting a chord length 2 units on 3x + 4y + 4 = 0**

**Sol.** Equation of the line is $3x + 4y + 4 = 0$

P = Length of the perpendicular

$= \frac{|3(-2) + 4.3 + 4|}{\sqrt{9+16}}$

$= \frac{10}{5} = 2$




Length of the chord $= 2\lambda = 2 \Rightarrow \lambda = 1$

If r is the radius of the circle then

$r^2 = 2^2 + 1^2 - 4 + 1 = 5$

Centre of the circle is (–2, 3)

Equation of the circle is $(x + 2)^2 + (y - 3)^2 = 5$

$x^2 + 4x + 4 + y^2 - 6y + 9 - 5 = 0$

i.e., $x^2 + y^2 + 4x - 6y + 8 = 0$

**12. If the two circles**
**$x^2 + y^2 + 2gx + 2fy = 0$ and**
**$x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other then show that f'g = fg'.**

**Sol.** $C_1 = (-g, -f)$ $\quad$ $C_2 = (-g^{-1}, f^{-1})$

$r_1 = \sqrt{g^2 + f^2}$ $\quad$ $r_2 = \sqrt{g'^2 + r'^2}$

$C_1 C_2 = r_1 + r_2$

$(C_1 C_2)^2 = (r_1 + r_2)^2$

$(g' - g)^2 + (f' - f)^2 = g^2 + f^2 + g'^2 + f'^2 + 2\sqrt{g^2 + f^2}\sqrt{g'^2 + r'^2}$

$-2(gg' + ff') = 2\{g^2g'^2 + f^2f'^2 + g^2f'^2 + f^2g'^2\}^{1/2}$

Squaring again

$(gg' + ff')^2 = g^2g'^2 + f^2f'^2 + g^2f'^2 + g'^2f^2$

$g^2g'^2 + f^2f'^2 + 2gg'ff' = g^2g'^2 + f^2f'^2 + g^2f'^2 + g'^2f'^2$

$2gg'ff' = g^2f'^2 + f^2g'^2$

$\Rightarrow g^2f'^2 + g'^2f^2 - 2gg'ff' = 0$

(or) $(gf' - fg')^2 = 0$

(or) $gf' = fg'$

**13. If P(x, y) be any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where S and S' are foci, then prove that SP + S'P is a constant.**

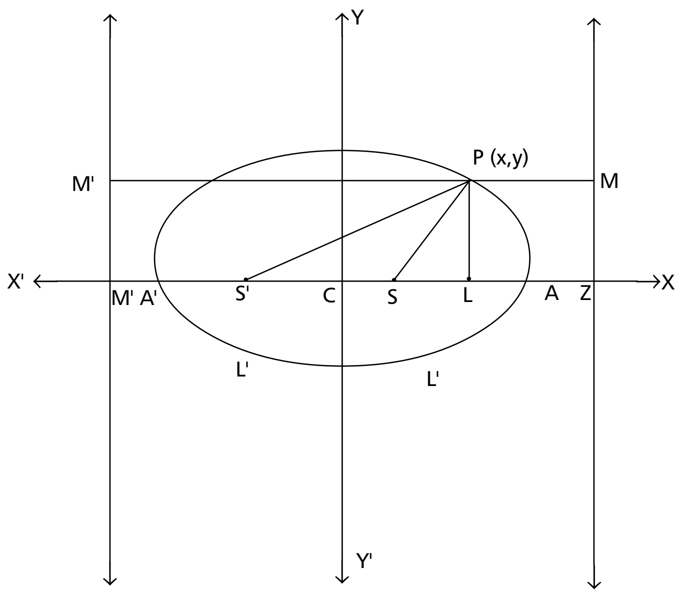
**Sol.** **Theorem :** If P (x, y) is any point on the ellipse $\frac{x^2+y^2}{a^2+b^2}=1$, (a > b). Whose foci are S and S' then SP + S'P is a constant.

**Proof :** The equation of the ellipse is given as

$$\frac{x^2+y^2}{a^2+b^2}=1, (a > b). \qquad ......(1)$$

Let S, S' be the foci and ZM, Z' M' be the corresponding directrices. Join SP and S'P. DRaw PL perpendicular to x-axis and M' MP perpendicular to the two directrices.

By the definition of the ellipse SP = ePM = e(LZ).




$$\therefore \text{SP} = e(\text{CZ} - \text{CL}) = e\left(\frac{a}{e} - x\right)$$

$$\therefore \text{SP} = a - xe$$

$\therefore$ S'P = ePM' = e(LZ')  Fig.

$$= e\left(x + \frac{a}{e}\right) = a + xe$$

$\therefore$ SP + S' P = a – xe + a + xe.

SP + S' P = 2a (constant) = Length of the major axis.

**14. *C* is the centrec, *AA*' and *BB*' are major and minor axis of the ellipse. $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1. If *PN* is the ordinate of a point *P* on the ellipse then show that $\frac{(PN)^2}{(A'N)\,(AN)} = \frac{(BC)^2}{(CA)^2}$**

**Sol.** Equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

P(a cos θ, b sin θ) any point on the ellipse.

PN = b sin θ; AN = a – a cos θ,

A'N = a + a cos θ ; BC = b, CA = a

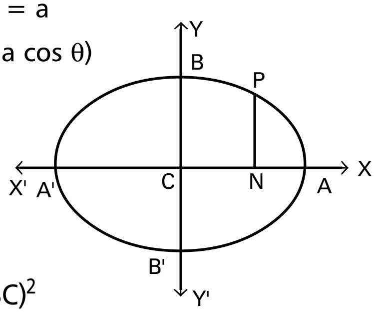
(A'N). (AN) = (a + a cos θ) (a – a cos θ)

$= a^2 - a^2 \cos^2 \theta.$

$= a^2 (1 - \cos^2 \theta) = a^2 \sin^2 \theta$

$$\frac{(PN)^2}{(A'N)\,(AN)} = \frac{b^2 \sin^2 \theta}{a^2 \sin^2 \theta} = \frac{b^2}{a^2}$$

$$\frac{BC^2}{(CA)^2} = \frac{b^2}{a^2} \Rightarrow \frac{PN^2}{(A'N)\,(AN)} = \frac{(BC)^2}{(CA)^2}$$




**15. One focus of a hyperbola is located at the point (1,– 3) and the corresponding directrix is the line y = 2. Find the equation of the hyperbola if its eccentricity is $\frac{3}{2}$.**

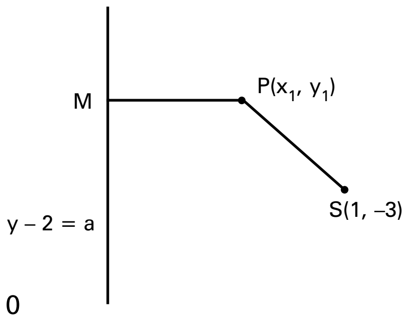
**Sol.** S(1 – 3) is the focus, equation of the directrix is y – 2 = 0

P($x_1$, $y_1$) is any point on the hyperbola join SP and draw PM perpendicular to the directrix S.P.= e. PM $\Rightarrow SP^2 = e^2 . PM^2$

$$(x_1 - 1)^2 + (y_1 + 3)^2 = \frac{9}{4}\left|\frac{(y_1 - 2)}{\sqrt{1+0}}\right|^2$$

$$x_1^2 + 1 - 2x_1 + y_1^2 + 9 + 6y_1$$

$$= \frac{9}{4}(y_1 - 2)^2$$




$$4x_1^2 + 4y_1^2 - 8x_1 + 24y_1 + 40$$

$$= 9\left(y_1^2 + 4 - 4y_1\right)$$

$$= 9y_1^2 - 36y_1 + 36$$

$$4x_1^2 - 5y_1^2 - 8x_1 + 60y_1 + 4 = 0$$

Focus of $P(x_1, y_1)$ is

$$4x^2 - 5y^2 - 8x + 60y + 4 = 0$$

This is the equation of the required hyperbola

**16. Evaluate** $\int_0^{2\pi} (1 + \cos x)^5 \ (1 - \cos x)^3 \, dx$ **.**

**Sol.** $\int_0^{2\pi} (1 + \cos x)^5 \ (1 - \cos x)^3 \, dx$ .

$$(1 + \cos x)^3 (1 + \cos x)^2 (1 - \cos x)^3$$

$$= \int_0^{2\pi} (1 - \cos^2 x)^3 (1 + \cos x)^2 \, dx = \int_0^{2\pi} (\sin^2 x)^3 (1 + \cos x)^2 \, dx$$

$$= \int_0^{2\pi} \sin^6 x \ (1 + \cos)^2 \, dx \quad = \cos x = t$$

$-\sin x \, dx = dt$

$\boxed{\sin x \, dx = -dt}$

U.L $\Rightarrow \cos 2\pi = t$

$\Rightarrow \boxed{1 = t}$

L.L $\Rightarrow \cos 0 = t$

$$\Rightarrow 1 = t = \int_{1}^{1} (1+t)^2 \, .dt$$

$$= \left(\frac{(1+t)^3}{3}\right)_1^1 = \frac{8}{3} - \frac{8}{3} = 0$$

**17. Solve $(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$**

**Sol.** Given $\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2} - \frac{x}{1+y^2}$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$$

$$P = \frac{1}{1+y^2}$$

$$\text{I.F.} = e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1} y}$$

$$x.\, e^{\tan^{-1} y} = \int e^{\tan^{-1} y} \cdot \frac{\tan^{-1} y}{1+y^2} \, dy$$

Put $t = \tan^{-1} y$ $dt = \frac{1}{1+y^2} dy$

$x\, .t = \int e^t .\, t\, dt = e^t(t - 1) + c$

$x\, .\, e^{\tan^{-1} y} = e^{\tan^{-1} y} (\tan^{-1} y - 1) + c$

is the solution.

## SECTION – C

**18. Show that the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ touch each other. Also find the point of contact and common tangent at this point of circle.**

**Sol.** Equations of the circles are

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

and $$x^2 + y^2 + 6x + 18y + 26 = 0$$

Centres are $C_1(2, 3)$, $C_2 = (-3, -9)$

$$r_1 = \sqrt{4 + 9 + 12} = 5$$

$$r_2 = \sqrt{9 + 81 - 26} = 8$$

$$C_1C_2 = \sqrt{(2 + 3)^2 + (3 + 9)^2}$$

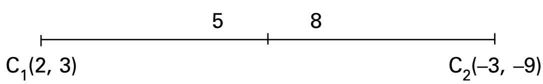
$$= \sqrt{25 + 144} = 13 = r_1 + r_2$$

$\therefore$ Circle touch externally

Equation of common tangent is $S_1 - S_2 = 0$

$$-10x - 24y - 38 = 0$$

$$5x + 12y + 19 = 0$$




The point of contact P divides $C_1C_2$ in the ratio 5 : 8

Co–ordinates of P are

$$\left(\frac{5(-3) + 8 \cdot 2}{5 + 8}, \frac{5(-9) + 8 \cdot 3}{5 + 8}\right) \Rightarrow \left(\frac{1}{13}, \frac{-21}{13}\right)$$

The common tangent at this point of contact is

$$x\left(\frac{1}{13}\right) + y\left(-\frac{21}{13}\right) - 2\left(x + \frac{1}{13}\right) - 3\left(y - \frac{21}{13}\right) - 12 = 0$$

i.e., $5x + 12y + 19 = 0$

**19. Find all common tangents of $x^2 + y^2 = 9$ and $x^2 + y^2 - 16x + 2y + 49 = 0$ the pairs of circles.**

**Sol.** The circles are $x^2 + y^2 = 9$

$$x^2 + y^2 - 16x + 2y + 49 = 0$$

Centre are A (0,0), B(8, –1)

$r_1 = 3$, $r_2 = \sqrt{64 + 1 - 49} = 4$

$AB = \sqrt{(0-8)^2 + (0+1)^2}$

$= \sqrt{64+1} = \sqrt{65} > r_1 + r_2$

The circles lie outside each other. A (0, 0), B (8, –1)

External center of similitude S divides AB externally in this ratio 3 : 4

Co–ordinates of are (–24, + 3)

Suppose m is the slope of the direct common tangents

$y - 3 = m(x + 24)$

$= mx + 24m$

$mx - y + (24m + 3) = 0$ —— (1)

This is a tangent to the circle $x^2 + y^2 = 9$

$$3 = \frac{|24m + 3|}{\sqrt{m^2+1}}$$

$9(m^2 + 1) = 9(8m + 1)^2$

$= 64m^2 + 10m + 1$

$63m^2 + 16m = 0$

$m(63m + 10) = 0$

$m = 0 \text{ or } \frac{-16}{63}.$

**Case (i) :** $m = 0$

Substituting in (1), equation of the tangent is $-y + 3 = 0$

$y - 3 = 0$

**Case (ii) :** $m = \frac{-16}{63}$

Equation of the tangent is $\frac{-16}{63}x - y + \left(\frac{-384}{63} + 3\right) = 0$

$\frac{-16}{63}x - y + \frac{195}{63} = 0$

$16x + 63y + 195 = 0$

Internal center of similitude $S^{I}$ divides AB internally in the ratio 3 : 4

Co-ordinates of $S^{I}$ are $\left(\frac{24}{7}, \frac{-3}{7}\right)$

Equation of the transverse common tangent is

$$y + \frac{3}{7} = m\left(x - \frac{24}{7}\right)$$

$$\frac{7y+3}{7} = \frac{m(7x-24)}{7}$$

$$7y + 3 = 7mx - 24m$$

$$7mx - 7y - (24m + 3) = 0 \qquad \text{—— (2)}$$

This is a tangent to the circle $x^2 + y^2 = 9$

$$3 = \frac{|24m+3|}{\sqrt{49m^2+49}} = \frac{3}{7}\frac{|28m+1|}{\sqrt{m^2+1}}$$

$$49(m^2 + 1) = (8m + 1)^2$$

$$49m^2 + 49 = 64m^2 + 16m + 1$$

$$15m^2 + 16m - 48 = 0$$

$$(3m - 4)(5m + 12) = 0$$

$$m = \frac{4}{3} \text{ or } \frac{-12}{5}$$

**Case (i) :** Substituting in (2), equation of the tangent is

$$\frac{28}{x}x - 7y - \left(\frac{96}{3} + 3\right) = 0$$

$$\frac{28}{x}x - 7y - \frac{105}{3} = 0$$

$$\frac{7}{3}(4x - 3y - 15) = 0$$

$$4x - 3y - 15 = 0$$

**Case (ii) :** $m = \frac{-12}{5}$

Equation of the transverse common tangent is

$$\frac{-84}{5}x - 7y - \left(\frac{-288}{5} + 3\right) = 0$$

$$\frac{-84}{5}x - 7y + \frac{273}{5} = 0$$

$$\frac{-7}{5}(12x + 5y - 39) = 0$$

ie $12x + 5y - 39 = 0$

$\therefore$ Equation of direct common tangents are

$y - 3 = 0$ $16x + 63y + 195 = 0$

Equation of transverse common tangent are

$4x - 3y - 15 = 0$ and $12x + 5y - 39 = 0$

**20. Find the equation of the parabola whose focus is S(3, 5) and vertex is A(1, 3).**

**Sol.** Equation of the axis $y - 3 = \frac{3-5}{1-3}(x - 1)$

$= x - 1$

$x - y + 2 = 0$

The directrix is perpendicular to the axis.

Equation of the directrix is $x + y + k = 0$

Co-ordinates of Z be (x, y)

A is the midpoint of SZ

Co-ordinates of A are $\left(\frac{3+x}{2}, \frac{5+y}{2}\right) = (1, 3)$

$\frac{3+x}{2} = 1$ $\qquad$ $\frac{5+y}{2} = 3$

$3 + x = 2$ $\qquad$ $5 + y = 6$

$x = 2 - 3 = -1$ $\qquad$ $y = 6 - 5 = 1$

Co-ordinates of Z are (–1, 1)

The directrix passes through Z (–1, 1)

$-1 + 1 + k = 0 \Rightarrow k = 0$

Equation of the directrix is $x - y = 0$

Equation of the parabola is $(x - \alpha)^2 + (y - \beta)^2$

$$= \frac{(lx + my + n)^2}{l^2 + m^2}$$

$$(x - 3)^2 + (y - 5)^2 = \frac{(x + y)^2}{1 + 1}$$

$\Rightarrow 2(x^2 - 6x + 9 + y^2 - 10y + 25) = (x + y)^2$

$\Rightarrow 2x^2 + 2y^2 - 12x - 20y + 68 = x^2 + 2xy + y^2$

**i.e.,** $x^2 - 2xy + y^2 - 12x - 20y + 68 = 0.$

**21. Evaluate $\int e^x \log (e^{2x} + 5e^x + 6)\, dx$ on R.**

**Sol.** $\int e^x \log (e^{2x} + 5e^x + 6)\, dx \quad \because e^{2x} + 5e^x + 6$

$= (e^x + 2)(e^x + 3)$

$= \int e^x . \log ((e^x + 2)(e^x + 3))\, dx$

$= \int e^x \{\log (e^x + 2) + \log (e^x + 3)\}\, dx$

$= \int e^x \log (e^x + 2)\, dx + \int e^x \log (e^x + 3)\, dx$

Put $e^x = t \Rightarrow e^x dx = dt$

$= \int \log (t + 2)\, dt + \int \log (t + 3)\, dt$

$= \log (t + 2) \int 1\, dt - \int \left\{\frac{d}{dt} \log (t+2). \int 1\, dt\right\} dt$

$+ \log(t + 3) \int 1\, dt - \int \left\{\frac{d}{dt} \log (t+3). \int 1\, dt\right\} dt$

$= t \log (t + 2) - \int \left(\frac{1}{t+2}\right) t\, dt + t \log (t + 3) - \int \frac{1}{t+3} \cdot t\, dt$

$= t \{\log (t + 2) + \log (t + 3)\} - \int \frac{t}{t+2}\, dt - \int \frac{t}{t+3}\, dt$

$= t \log (t^2 + 5t + 6) - \int \left(\frac{t+2-2}{t+2}\right) dt - \int \left(\frac{t+3-3}{t+3}\right) dt$

$= t. \log (t^2 + 5t + 6) - \int \left\{1 - \frac{2}{t+2}\right\} dt - \int \left\{1 - \frac{3}{t+3}\right\} dt$

$= t \log (t^2 + 5t + 6) - t + 2 \log |t + 2| - t + 3 \log |t + 3| + C$

$= t . \log (t^2 + 5t + 6) - 2t + 2 \log |t + 2| + 3 \log |t + 3| + C$

$= e^x \log (e^{2x} + 5e^x + 6) - 2e^x + 2 \log (e^x + 2) + 3 \log (e^x + 3) + C$

**22. Find** $\int \frac{2x+1}{x(x^2+4)^2}\,dx.$

**Sol.** Let $\frac{2x+1}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+c}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$

$2x + 1 = A\,(x^2 + 4)^2 + (Bx + C) + x\,(x^2 + 4) + (Dx + E)\,x$

Equating the coefficients of like power of x, we obtain

$A + B = 0,\ C = 0,\ 8A + 4B + D = 0,$

$$4C + E = 2,\ A = \frac{1}{16}$$

Solving these equations, we obtain

$A = \frac{1}{16},\ B = -\frac{1}{16},\ C = 0,\ D = -\frac{1}{4},\ E = 2$

$$\int \frac{2x+1}{x(x^2+4)^2}dx = \frac{1}{16}\int\frac{dx}{x} - \frac{1}{32}\int\frac{2x}{x^2+4}dx + \int\frac{\left(-\frac{1}{4}x+2\right)}{(x^2+4)^2}dx \quad .... (1)$$

$\int \frac{dx}{(x^2+4)^2}$

Put $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta\, d\theta$

$$\int\frac{dx}{(x^2+4)^2} = \int\frac{2\sec^2\theta\, d\theta}{16\,(1+\tan^2\theta)^2} = \frac{1}{8}\int\frac{d\theta}{\sec^2\theta} = \frac{1}{8}\int\cos^2\theta\, d\theta$$

$$= \frac{1}{16}\int(1 + \cos 2\theta)\, d\theta = \frac{1}{16}\left(\theta + \sin\frac{2\theta}{2}\right)$$

$$= \frac{1}{16}\left(\theta + \frac{\tan\theta}{1+\tan^2\theta}\right) + C_2 = \frac{1}{16}\left[\tan^{-1}\left(\frac{x}{2}\right) + \frac{2x}{4+x^2}\right] + C_2$$

From (1) and (2) we get

$$\int\frac{2x+1}{x(x^2+4)^2}dx = \frac{1}{16}\log|x|$$

$$-\frac{1}{32}\log(x^2+4) + \frac{1}{8(x^2+4)} + \frac{1}{8}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{4}\left(\frac{x}{4+x^2}\right) + C$$

**23. Show that the area enclosed between the curve $y^2 = 12(x + 3)$ and $y^2 = 20(5 - x)$ is $64\sqrt{\frac{5}{3}}$.**

**Sol.** Equation of the curves are

$y^2 = 12(x + 3)$ —— (1)

$y^2 = 20(5 - x)$ —— (2)

Eliminating y

$12(x + 3) = 20(5 - x)$

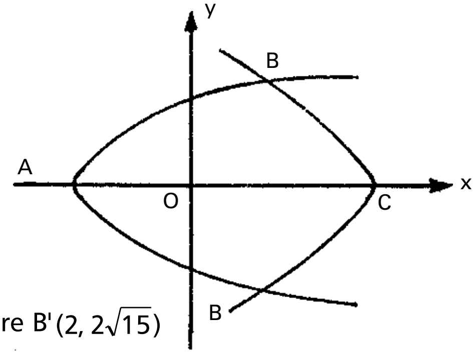
$3x + 9 = 25 - 5x$

$8x = 16$

$x = 2$

$y^2 = 12(2 + 3) = 60$

$y = \sqrt{60} = \pm 2\sqrt{15}$




Points of intersection are B' $(2, 2\sqrt{15})$

B' $(+2, -2\sqrt{15})$

The required area is symmetrical about X – axis

Area ABCB'

$$= 2\left[\int_{-3}^{2} 2\sqrt{3}\sqrt{x+3}\,dx + \int_{2}^{5} 2\sqrt{5}\sqrt{5-x}\,dx\right]$$

$$= 4\sqrt{3}\left(\frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}}\right)_{-3}^{2} + 4\sqrt{5}\left(\frac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}}\right)_{2}^{5}$$

$$= \frac{8\sqrt{3}}{3}\left(5^{\frac{3}{2}} - 0\right) - \frac{8\sqrt{5}}{3}\left[0 - 3^{\frac{3}{2}}\right]$$

$$= \frac{8\sqrt{3}}{3}.5\sqrt{5} + \frac{8\sqrt{5}}{3}.3\sqrt{3} = \frac{40.\sqrt{15}}{3} + \frac{24\sqrt{15}}{3}$$

$$= \frac{64}{3}\sqrt{15} \text{ sq. units} = 64\sqrt{\frac{15}{9}} \text{ sq. units} = 64\sqrt{\frac{5}{3}} \text{ sq. units.}$$

**24. Solve** $\dfrac{dy}{dx} = \dfrac{-(x^2+3y^2)}{3x^2+y^2}$

**Sol.** $\dfrac{dy}{dx} = \dfrac{-(x^2+3y^2)}{(3x^2+y^2)}$

Put y = vx

$$\frac{dy}{dx} = v + x \,.\, \frac{dv}{dx}$$

$$v + x \,.\, \frac{dv}{dx} = \frac{-(x^2+3v^2x^2)}{3x^2+v^2x^2}$$

$$= \frac{-x^2(1+3v^2)}{x^2(3+v^2)}$$

$$x \,.\, \frac{dv}{dx} = -v - \frac{1+3v^2}{3+v^2}$$

$$= \frac{-3v - v^3 - 1 - 3v^2}{3+v^2} = -\frac{(v+1)^3}{3+v^2}$$

$$\frac{3+v^2}{(v+1)^3} = \frac{-dx}{x}$$

$$\frac{3+v^2}{(v+1)^3} = \frac{A}{v+1} + \frac{B}{(v+1)^2} + \frac{C}{(v+1)^3}$$

Multiplying with $(v + 1)^3$

$3 + v^2 = A(v + 1)^2 + B(v + 1) + C$

$v = -1 \Rightarrow 3 + 1 = C \Rightarrow C = 4$

Equating the co-efficients of $v^2$

$A = 1$

Equating the co-efficients of v

$0 = 2A + B$

$B = -2A = -2$

$$\frac{v^2+3}{(v+1)^3} = \frac{1}{v+1} - \frac{2}{(v+1)^2} + \frac{4}{(v+1)^3}$$

$$\int \frac{v^2 + 3}{(v + 1)^3} = -\int \frac{dx}{x}$$

$$\int \left( \frac{1}{v + 1} - \frac{2}{(v + 1)^2} + \frac{4}{(v + 1)^3} \right) dv = -\log x + \log c$$

$$\log (v + 1) + \frac{2}{v + 1} - \frac{4}{2(v + 1)^2} = \log \frac{c}{x}$$

Solution is $\log\left(\frac{y}{x} + 1\right) + \frac{2}{\frac{y}{x} + 1} - \frac{2}{\left(\frac{y}{x} + 1\right)^2} = \log\frac{c}{x}$

$$\frac{2x}{x + y} - \frac{2x^2}{(x + y)^2} = \log\frac{c}{x} - \log \frac{(x + y)}{x}$$

$$\frac{2x^2 + 2xy - 2x^2}{(x+y)^2} = \log\frac{c}{x+y}$$

Solution is $\frac{2xy}{(x + y)^2} = \log\frac{c}{x + y}$

Solution is $\log \left(\frac{x + y}{c}\right) c = -\log\left(\frac{c}{x + y}\right)$

$$= -\frac{2xy}{(x + y)^2}$$

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