PRACTICE PAPER – 5

SOLUTIONS

SECTION – A

I. 1. If the length of the tangent from (2, 5) to the circle $x^2 + y^2 - 5x + 4y + k = 0$ is $\sqrt{37}$ then find k.

Sol. Length of tangent = $\sqrt{S_{11}}$

$= \sqrt{(2)^2 + (5)^2 - 5 \times 2 + 4 \times 5 + k} = 37 = 39 + k$

k = – 2 units.

2. $x^2 + y^2 - 4x - 6y - 12 = 0$, $x^2 + y^2 + 6x + 18y + 26 = 0$ find the relative position of the pair of circles.

Sol. Centres of the circles are A (2,3), B(–3, – 9)

radii are $r_1 = \sqrt{4+9+12} = 5$

$r_2 = \sqrt{9+81-26} = 8$

$AB = \sqrt{(2+3)^2 + (3+9)^2} = \sqrt{25+144} = 13 = r_1 + r_2$

∴ The circle touch externally.

3. Show that the circles given by the following equations intersect each other orthogonally.

$x^2 + y^2 - 2x - 2y - 7 = 0$, $3x^2 + 3y^2 - 8x + 29y = 0$.

Sol. $C_1 = (1, 1)$

$g = -1, f = -1, c = -7; g' = \frac{-4}{3}, f' = \frac{29}{6}; c' = 0$

Condition that two circles are orthogonal is 2gg' + 2ff' = c + c'

$2(-1)\left(\frac{-4}{3}\right) + 2(-1)\,\frac{29}{6} = -7 + 0$

L.H.S. $= \frac{8}{3} - \frac{29}{3} = \frac{-21}{3} = -7$

$-7 = -7$

Hence both circles cut orthogonally.

4. Find the position (interior or exterior or on) of the points (6, –6) with respect to the parabola $y^2 = 16x$.

Sol. Equation of the parabola is $y^2 = 6x$

i.e., $S \equiv y^2 - 6x$

$S_{11} = (-6)^2 - 6.6 = 36 - 36 = 0$

$\therefore$ (6, – 6) lies on the parabola.

5. Find the equation of the normal at $\theta = \frac{\pi}{3}$ to the hyperbola $3x^2 - 4y^2 = 12$.

Sol. Equation of the hyperbola is $3x^2 - 4y^2 = 12$

$$\frac{x^2}{4} - \frac{y^2}{3} = 1$$

Equation of the normal is $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$

$$\frac{2x}{\sec 60^o} + \frac{\sqrt{3}y}{\tan 60^o} = 4 + 3$$

$$\frac{2x}{2} + \frac{\sqrt{3}y}{\sqrt{3}} = 7 \Rightarrow x + y = 7$$

6. $\int \frac{1+\cos^2 x}{1-\cos 2x} dx$ on $I \subset R / \{n\pi : n \in z\}$.

Sol. $\int \frac{1+\cos^2 x}{1-\cos 2x} dx = \int \frac{1+\cos^2 x}{2\sin^2 x} dx$

$$= \frac{1}{2}\int \frac{dx}{\sin^2 x} + \frac{1}{2}\int \cot^2 x \, dx$$

$$= \frac{1}{2}\int \operatorname{cosec}^2 x\,dx + \frac{1}{2}\int (\operatorname{cosec}^2 x - 1)\,dx$$

$$= \int \operatorname{cosec}^2 x\,dx - \frac{1}{2}\int dx = -\cot x - \frac{x}{2} + C$$

7. Evaluate $\int \frac{1}{x\log x\,[\log(\log x)]}\,dx$ on $(1, \infty)$.

Sol. $t = \log(\log x)$

$$dt = \frac{1}{\log x}\cdot\frac{1}{x}\,dx$$

$$\int \frac{1}{x\log x\,[\log(\log x)]}\,dx = \int \frac{dt}{t}$$

$$= \log|t| + C = \log|\log(\log x)| + C$$

8. Evaluate $I = \int_0^1 \frac{x^2}{x^2+1}\,dx$.

Sol. $= \int_0^1 \frac{(x^2+1-1)}{x^2+1}\,dx$

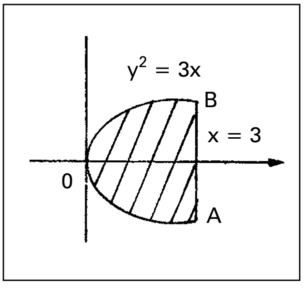
$$= \int_0^1 dx - \int_0^1 \frac{dx}{x^2+1} = [x]_0^1 - \left[\tan^{-1} x\right]_0^1$$

$$= 1 - \tan^{-1} 1 = 1 - \pi/4$$

9. Find the area enclosed with the curves $y^2 = 3x$, $x = 3$.

Sol. The parabola is symmetrical about X – axis

Required area $= 2\int_0^3 \sqrt{3}.\sqrt{x}\,dx$



$$= \left(2\sqrt{3}\,.\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^3$$

$$= \frac{4\sqrt{3}}{3}\,.\,(3\sqrt{3} - 0) = 12 \text{ sq. units.}$$

10. Find the order of the differential equation of the family of all circles with their centres at the origin.

Sol. Equation of the circle with centre at origin is $x^2 + y^2 = r^2$

Order = no .of arbitrary constants = 1

SECTION – B

II.11. Find the equation of the circum circle of the triangle formed by the straight lines given in each of the following.

x + 3y – 1 = 0, x + y + 1 = 0, 2x + 3y + 4 = 0.

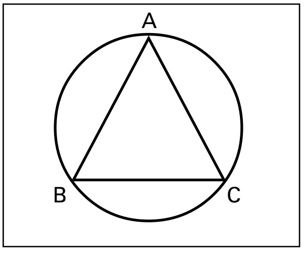
Sol. AB : x +3y – 1 = 0 AB : x + 3y –1 = 0

AC : x + y +1 = 0 AC : x + y + 1 = 0

A : (1 – 2) B : (– 5, 2)

BC : 2x + 3y + 4 = 0 BC : 2x + 3y + 4 = 0

C : (– 2, 1)



Equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$

A, B, C are points on circumference.

$\therefore$ 1 + 4 + 2g – 4f + c = 0 — (i)

25 + 4 – 10g + 4f + c = 0 — (ii)

4 + 1– 4g + 2f + c = 0 — (iii)

Subtracting (i) – (iii) we get

6g – 6f = 0 (or) g = f — (iv)

Subtracting (i) – (ii) we get

24 – 12g + 8f = 0 — (v)

Solving (iv) and (v) we get

$g = 6, f = 6, c = 7$

Required equation of circle be

$x^2 + y^2 + 12x + 12y + 7 = 0$

12. Find the equation of the circle which cuts the following circles orthogonally.

$x^2 + y^2 + 2x + 4y + 1 = 0, 2x^2 + 2y^2 + 6x + 8y - 3 = 0$

$x^2 + y^2 - 2x + 6y - 3 = 0.$

Sol. Equations of the required circles are

$S \equiv x^2 + y^2 + 2x + 4y + 1 = 0$

$S_1 \equiv x^2 + y^2 + 3x + 4y - \frac{3}{2} = 0$

$S_{11} \equiv x^2 + y^2 - 2x + 6y - 3 = 0$

Radical axis of $S = 0, S_1 = 0$ is $S - S_1 = 0 - x + \frac{5}{2} = 0 \Rightarrow x = \frac{5}{2}$

Radical axis of $S = 0, S_{11} = 0$ is $S - S_{11} = 0$

$4x - 2y + 4 = 0 \Rightarrow 2x - y + 2 = 0$

$x = \frac{5}{2} \Rightarrow 5 - y + 2 = 0 \Rightarrow y = 7$

Radical centre is $P\left(\frac{5}{2}, 7\right)$

PT = Length of the tangent from P to S = 0

$= \sqrt{\frac{25}{4} + 49 + 5 + 28 + 1}$

$= \sqrt{\frac{25}{4} + 83} = \sqrt{\frac{25 + 332}{4}} = \frac{\sqrt{357}}{2}$

Equation of the circles cutting the given circles orthogonally

$\left(x - \frac{5}{2}\right)^2 + (y - 7)^2 = \frac{357}{4}$

$$x^2 - 5x + \frac{25}{4} + y^2 - 14y + 49 = \frac{357}{4}$$

$$x^2 + y^2 - 5x - 14y + \frac{25}{4} + 49 - \frac{357}{4} = 0$$

$$x^2 + y^2 - 5x - 14y + \frac{25 + 196 - 357}{4} = 0$$

$$x^2 + y^2 - 5x - 14y - \frac{136}{4} = 0$$

$$x^2 + y^2 - 5x - 14y - 34 = 0$$

13. Prove that the product of the perpendicular distance from any point on the hyperbola to its asymptotes is constant.

Sol. Equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Any point on the hyperbola is $P(a\sec\theta, b\tan\theta)$

Equations of the asymptotes are $\frac{x}{a} = \pm\frac{y}{b}$

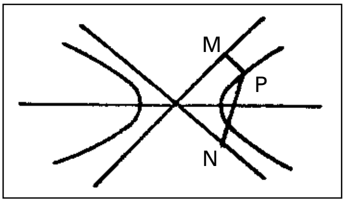
i.e., $\frac{x}{a} - \frac{y}{b} = 0$ and $\frac{x}{a} + \frac{y}{b} = 0$

PM = Perpendicular distance from P on $\frac{x}{a} - \frac{y}{b} = 0$

$$= \frac{|\sec\theta - \tan\theta|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

PN = Perpendicular distance from P on

$\frac{x}{a} + \frac{y}{b} = 0$



$$= \frac{|\sec\theta + \tan\theta|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

$$PM.\ PN = \frac{|\sec\theta - \tan\theta|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \frac{|\sec\theta + \tan\theta|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

$$= \frac{|\sec^2\theta - \tan^2\theta|}{\left(\frac{1}{a^2}+\frac{1}{b^2}\right)} = \frac{1}{\frac{a^2+b^2}{a^2b^2}} \quad \frac{a^2b^2}{a^2+b^2} = \text{constant}$$

14. Find the equation of tangents to the ellipse $2x^2 + y^2 = 8$ which are :

i) Parallel to $x - 2y - 4 = 0$

Sol. Slope will be : $\frac{1}{2}$

Equation of tangent $y = mx \pm \sqrt{a^2m^2 + b^2}$

$$y = \frac{1}{2}x \pm \sqrt{a^2\left(\frac{1}{2}\right)^2 + b^2} \ ; \ \frac{x^2}{4} + \frac{y^2}{8} = 1$$

$$y = \frac{1}{2}\times \pm \sqrt{4\times\frac{1}{4}+8} \ ; \ y = \frac{1}{2}\times \pm 3$$

$2y - x \pm 6 = 0$ required equation of tangents.

$x - 2y \pm 6 = 0.$

15. Evaluate $\int x^2 \sin^{-1} x\, dx, x \in (-1, 1).$

Sol. $\int x^2 \sin^{-1} x\, dx = (\sin^{-1} x)\ \frac{x^3}{3} - \frac{1}{3}\int x^3 \left(\frac{1}{\sqrt{1-x^2}}\right) dx$

$$= \frac{x^3}{3}\ \sin^{-1} x - \frac{1}{3}\int \frac{x[1-(1-x^2)]}{\sqrt{1-x^2}}\ dx$$

$$= \frac{x^3}{3}\sin^{-1}x - \frac{1}{3}\int \frac{x\, dx}{\sqrt{1-x^2}} + \frac{1}{3}\int x\sqrt{1-x^2}\ dx$$

$$= \frac{x^3}{3}\sin^{-1}x + \frac{1}{3}\sqrt{1-x^2} + \frac{1}{3}\ \frac{\left(1-x^2\right)^{\frac{3}{2}}}{\frac{3}{2}(-2)} + C$$

$$= \frac{x^3}{3}\ \sin^{-1} x + \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}(1-x^2)^{3/2} + C.$$

16. Solve $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$.

Sol. $I = \int_0^{\pi} \frac{x}{1+\sin x}\,dx$ —— (i)

$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{1+\sin(\pi - x)}$ —— (ii)

$2I = \int_0^{\pi} \frac{\pi\,dx}{1+\sin x}$

$I = \frac{\pi}{2}\int_0^{\pi} \frac{dx}{1+\sin x}$

$= \frac{\pi}{2}\int_0^{\pi} \frac{(1-\sin x)}{1-\sin^2 x}dx = \frac{\pi}{2}\int_0^{\pi}\left(\frac{1-\sin x}{\cos^2 x}\right)dx$

$= \frac{\pi}{2}\left(\int_0^{\pi}\frac{1}{\cos^2 x}\,dx - \int_0^{\pi}\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx\right)$

$= \frac{\pi}{2}\int_0^{\pi}\sec^2 x\,dx - \int_0^{\pi}\sec x\,.\tan x\,dx$

$= \frac{\pi}{2}\left(\left[\tan x\right]_0^{\pi} - \left[\sec x\right]_0^{\pi}\right)$

$= \frac{\pi}{2}[(0-0)-(-1-1)] = \frac{\pi}{2}\,.\,2 = \pi$

17. Solve $\sin^{-1}\left[\frac{dy}{dx}\right] = x + y$.

Sol. $\frac{dy}{dx} = \sin(x+y)$

$x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{dt}{dx} - 1 = \sin t$$

$$\frac{dt}{dx} = 1 + \sin t$$

$$\frac{dt}{1 + \sin t} = dx$$

Integrating both sides we get

$$\int \frac{dt}{1 + \sin t} = \int dx$$

$$\int \frac{1 - \sin t}{\cos^2 t}\, dt = x + c$$

$$\int \sec^2 t\, dt - \int \tan t \,.\, \sec t\, dt = x + c$$

$\tan t - \sec t = x + c \Rightarrow \tan (x + y) - \sec (x + y) = x + c$

SECTION – C

III.18. Find the equation of the circle which passes through the vertices of the triangle formed by $L_1 = x + y + 1 = 0$, $L_2 = 3x + y - 5 = 0$, $L_3 = 2x + y - 5 = 0$.

Sol. Suppose L_1, L_2; L_2, L_3 and L_3, L_1 intersect in A, B and C respectively.

Consider a curve whose equation is

$k (x + y + 1) (3x + y - 5) + l(3x + y - 5)$

$(2x + y - 5) + m(2x + y - 5) (x + y + 1) = 0$ —— (1)

This equation represents a circle

i) Co-efficient of x^2 = Co – efficient of y^2

$3k + 6l + 2m = k + l + m$

$2k + 5l + m = 0$ —— (2)

ii) Co-efficient of xy = 0

$4k + 5l + 3m = 0$ —— (3)

Applying cross multiplication rule for (2) and (3) we get

$$\begin{matrix} & k & & l & & m & \\ 5 & \times & 1 & \times & 2 & \times & 5 \\ 5 & & 3 & & 4 & & 5 \end{matrix}$$

$$\frac{k}{15-5} = \frac{l}{4-6} = \frac{m}{10-20}$$

$$\frac{k}{10} = \frac{l}{-2} = \frac{m}{-10} \Rightarrow \frac{k}{5} = \frac{l}{-1} = \frac{m}{-5}$$

Substituting in (1), equation of the required circle is

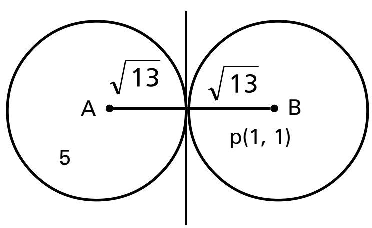
$5(x + y + 1)(3x + y - 5) - 1(3x + y - 5)$

$(2x + y - 5) - 5(2x + y - 5)(x + y + 1) = 0$

i.e., $x^2 + y^2 - 30x - 10y + 25 = 0$

19. Find the equations of circles which touch $2x - 3y + 1 = 0$ at (1, 1) and having radius $\sqrt{13}$.

Sol. The centres of required circle lies on a line perpendicular to $2x - 3y + 1 = 0$ and passing through (1, 1)



The equation of the line of centre can be taken as

$3x + 2y + k = 0$

This line passes through (1, 1)

$3 + 2 + k = 0 \Rightarrow k = -5$

Equation of AB is $3x + 2y - 5 = 0$

The centres A and B are situated on

$3x + 2y - 5 = 0$ at a distance $\sqrt{13}$ from (1, 1).

The centre B are given by $(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$

$$\left(1+\sqrt{13}\left(-\frac{2}{\sqrt{13}}\right), 1+\sqrt{13}.\frac{3}{\sqrt{13}}\right) \text{ and}$$

$$\left(1-\sqrt{13}\,\frac{(-2)}{\sqrt{13}}, 1-\sqrt{13}.\frac{3}{\sqrt{13}}\right)$$

i.e., (1 – 2, 1 + 3) and (1 + 2, 1 – 3) (– 1, 4) and (3, – 2)

Centre (3, –2), r = $\sqrt{13}$

Equation of the required circles are

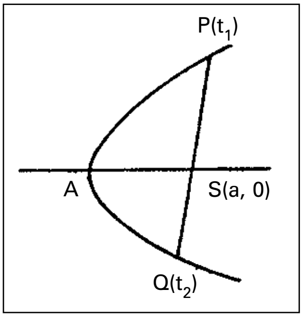
$(x + 1)^2 + (y - 4)^2 = 13$ and $(x - 3)^2 + (y + 2)^2 = 13$

i.e., $x^2 + y^2 + 2x - 8y + 4 = 0$ and $x^2 + y^2 - 6x + 4y = 0$

20. Show that the tangent at one extremity of a focal chord of a parabola is parallel to the normal at the other extremity.

Sol. $P(t_1)$, $Q(t_2)$ are the ends of a focal chord.

Slope of PS = Slope of PQ



$$\frac{2at_1}{a(t_1^2-1)} = \frac{2a(t_1-t_2)}{a(t_1^2-t_2^2)}$$

$$\frac{t_1}{t_1^2-1} = \frac{1}{t_1+t_2}$$

$$t_1 + t_2 = \frac{t_1^2-1}{t_1} = t_1 - \frac{1}{t_1}$$

$$t_2 = -\frac{1}{t_1} \qquad \text{—— (1)}$$

Equation of the tangent at $P(t_1)$ is $t_1y = x + at_1^2$

Slope of the tangent at P = $\frac{1}{t_1}$ —— (2)

Equation of the normal at $Q(t_2)$ is $y + xt_2 = 2at_2 + at_2^3$

Slope of the normal at Q = $-t_2$ —— (3)

From (1), (2), (3) we get slope of the tangent at P = slope of normal at Q

Slope of the tangent at P is parallel to the normal at Q.

21. Solve $\int (6x+5)\sqrt{6-2x^2+x}\,dx.$

Sol. Let 6x + 5 = A (1 – 4x) + B

Equating the co-efficients of x; $6 = -4A \Rightarrow A = \frac{-3}{2}$

Equating the constants A + B = 5; $B = 5 - A = 5 + \frac{3}{2} = \frac{13}{2}$

$$\int (6x+5)\sqrt{6-2x^2+x}\,dx$$

$$= -\frac{3}{2}\int (1-4x)\sqrt{6-2x^2+x}\,dx + \frac{13}{2}\int\sqrt{6-2x^2+x}\,dx$$

$$= -\frac{3}{2}\,\frac{(6-2x^2+x)^{3/2}}{\frac{3}{2}} + \frac{13}{2}.\sqrt{2}\int\sqrt{3-x^2+\frac{x}{2}}\,dx$$

$$= -(6-2x^2+x)^{3/2} + \frac{13}{\sqrt{2}}\int\sqrt{\left(\frac{7}{4}\right)^2-\left(x-\frac{1}{4}\right)^2}\,dx$$

$$= -(6-2x^2+x)^{3/2} + \frac{13}{\sqrt{2}}\left(\frac{\left(x-\frac{1}{4}\right)\sqrt{3-x^2+\frac{x}{2}}}{2} + \frac{49}{32}\sin^{-1}\frac{\left(x-\frac{1}{4}\right)}{\left(\frac{7}{4}\right)}\right) + C$$

$$= -(6-2x^2+x)^{3/2} + \frac{13}{\sqrt{2}}\,\frac{(4x-1)\sqrt{(6-2x^2+x)}}{16\quad 2} + \frac{49}{32}\sin^{-1}\left(\frac{4x-1}{7}\right) + C$$

$$= (6-2x^2+x)^{3/2} + \frac{13}{16}(4x-1)\sqrt{6-2x^2+x} + \frac{637}{32\sqrt{2}}\sin^{-1}\left(\frac{4x-1}{7}\right) + C$$

22. If $I_n = \int (\log x)^n .dx$, then show that $I_n = x (\log x)^n - n\, I_{n-1}$ and hence find $\int (\log x)^4\, dx$.

Sol. $I_n = \int (\log x)^n\, dx$

$= (\log x)^n . x - \int x . n . (\log x)^{n-1} . \frac{1}{x}\, dx$

$= x. (\log x)^n - n \int (\log x)^{n-1}\, dx$

$= x (\log x)^n - n . I_{n-1}$

$I_4 = x(\log x)^4 - 4 . I_3$

$I_3 = (x \log x)^3 - 3 . I_2$

$I_2 = (x \log x)^2 - 2. I_1$

$I_1 = x \log x - I_0$ where $I_0 = \int dx = x$

$I_1 = x \log x - x$

$I_2 = (x (\log x)^2 - 2x \log x + 2x$

$I_3 = (x (\log x)^3 - 3(x (\log x)^2 - 2x \log x + 2x)$

$= x . (\log x)^3 - 3x (\log x)^2 + 6x (\log x) - 6x$

$I_4 = x(\log x)^4 - 4[x. (\log x)^3 - 3x (\log x)^2 + 6x (\log x) - 6x] + C$

$= x [(\log x)^4 - 4(\log x)^3 + 12(\log x)^2 - 24 (\log x) + 24] + C$

23. Evaluate $\int_0^{\pi/2} \frac{\sin^2 x}{\cos x + \sin x} dx.$

Sol. $I = \int_0^{\pi/2} \frac{\sin^2 x\, dx}{\cos x + \sin x} = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx$

$= \int_0^{\pi/2} \frac{\cos^2 x\, dx}{\sin x + \cos x} = \frac{1}{2} \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx$

$$= \frac{1}{2}\int_0^{\pi/2} \frac{dx}{\sin x + \cos x} \quad \text{——— (1)}$$

Consider $\int_0^{\pi/2} \frac{dx}{\sin x + \cos x}$

Put $\tan\frac{x}{2} = t$

$dx = \frac{2dt}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$, $\sin x = \frac{2t}{1+t^2}$

$$\int_0^{\pi/2} \frac{dx}{\sin x + \cos x} = \int_0^1 \frac{2t\,dt}{2t + (1-t^2)}$$

$$= 2\int_0^1 \frac{dt}{(\sqrt{2})^2 - (t-1)^2} = 2.\frac{1}{2\sqrt{2}}\left[\log\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right]_0^1$$

$$= \frac{1}{\sqrt{2}}\left(\log 1 - \log\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \frac{1}{\sqrt{2}}.\log\frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)^2 = \frac{2}{\sqrt{2}}\log(\sqrt{2}+1)$$

$$I = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$$

24. Solve $\frac{dy}{dx}(x^2y^3 + xy) = 1$.

Sol. $\frac{dx}{dy} = xy + x^2y^3$

This is Bernoulli's equation

$$x^{-2}.\frac{dx}{dy} - \frac{1}{x}.y = y^3$$

Put $z = -\frac{1}{x}$ so that $\frac{dz}{dy} = \frac{1}{x^2}\frac{dx}{dy}$

$$\frac{dz}{dy} + z.\,y = y^3$$

$$\text{I.F.} = e^{\int y\,dy} = e^{\frac{y^2}{2}}$$

$$z.\,e^{y^2/2} = \int y^3.\,e^{\frac{y^2}{2}}.\,dy$$

Consider $\int y^3.\,e^{\frac{y^2}{2}}.\,dy$

$$t = \frac{y^2}{2} \Rightarrow dt = y\,dy$$

$$\int y^3.\,e^{y^2}.\,dy = \int t.\,dt.\,e^t = e^t(t-1)$$

$$= e^{\frac{y^2}{2}}\left(\frac{y^2}{2} - 1\right)$$

$$z.\,e^{\frac{y^2}{2}} = e^{\frac{y^2}{2}}\left(\frac{y^2}{2} - 1\right) + c$$

$$z = \frac{y^2}{2} - 1 + c.\,e^{-\frac{y^2}{2}}$$

$$-\frac{1}{x} = \frac{y^2}{2} - 1 + c.\,e^{-\frac{y^2}{2}}$$

$$-1 = x\left(\frac{y^2}{2} - 1 + c.\,e^{-\frac{y^2}{2}}\right)$$

Solution is $1 + x\left(\frac{y^2}{2} - 1 + c.\,e^{-\frac{y^2}{2}}\right) = 0$

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